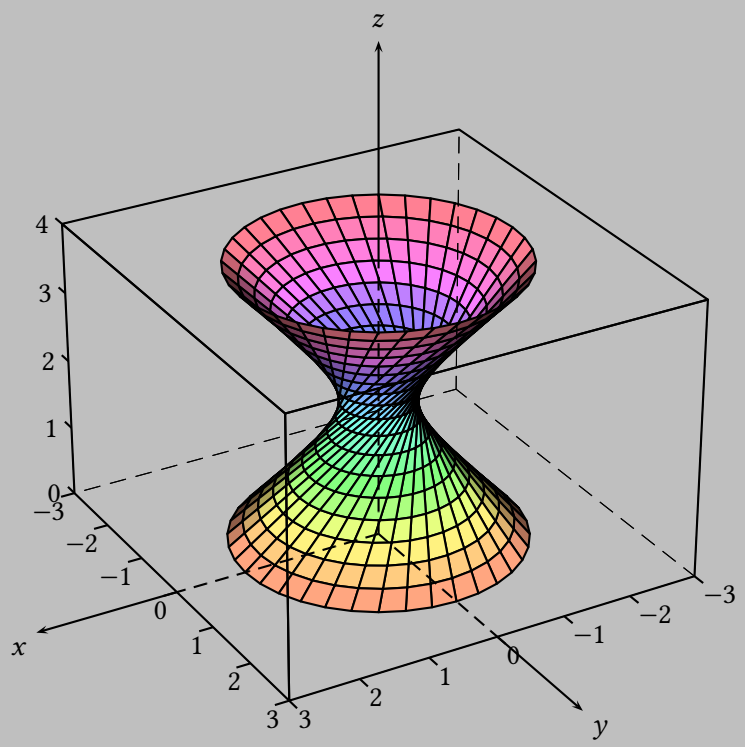
PSTricks

Hyperbolids with PSTricks

June 4, 2026



Package author(s):
Manuel Luque
Herbert Voß

Contents

1 Introduction

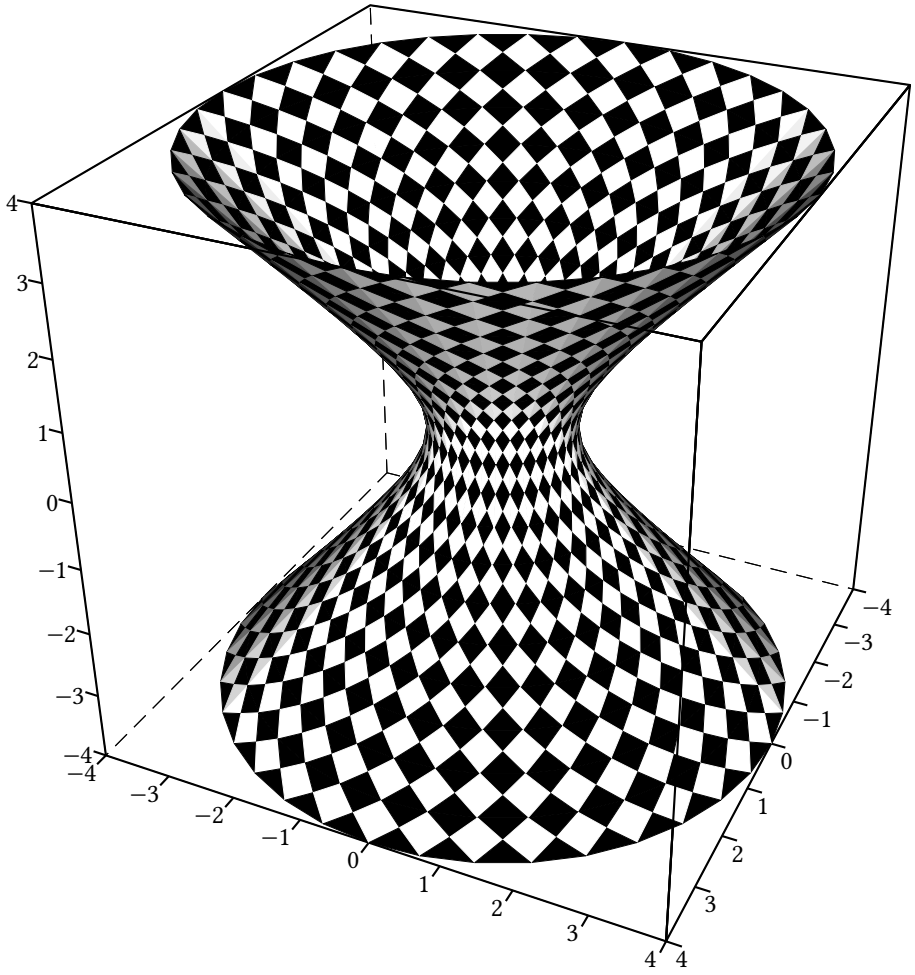
The command is `\psHyperboloid[`*options*`](x,y,z)`. The examples can be run with the sequence `latex→""dvips→""ps2p` or with `lualatex` if you want directly the `.pdf` output. Some of the examples with a splitted object needs heavy calculations which may take more than 30 minutes on slow machines!

The mesh is defined by the option `ngrid=n1 n2`, where `n1` is the number of meridians and `n2` is the number of levels. The height of the hyperboloid is defined by the option `h=8`, and the radius of the bases by `R=4` (default values). The number of inner and outer facets is calculated using the formula:

$$n_1 \times n_2 \times 4 \times 2$$

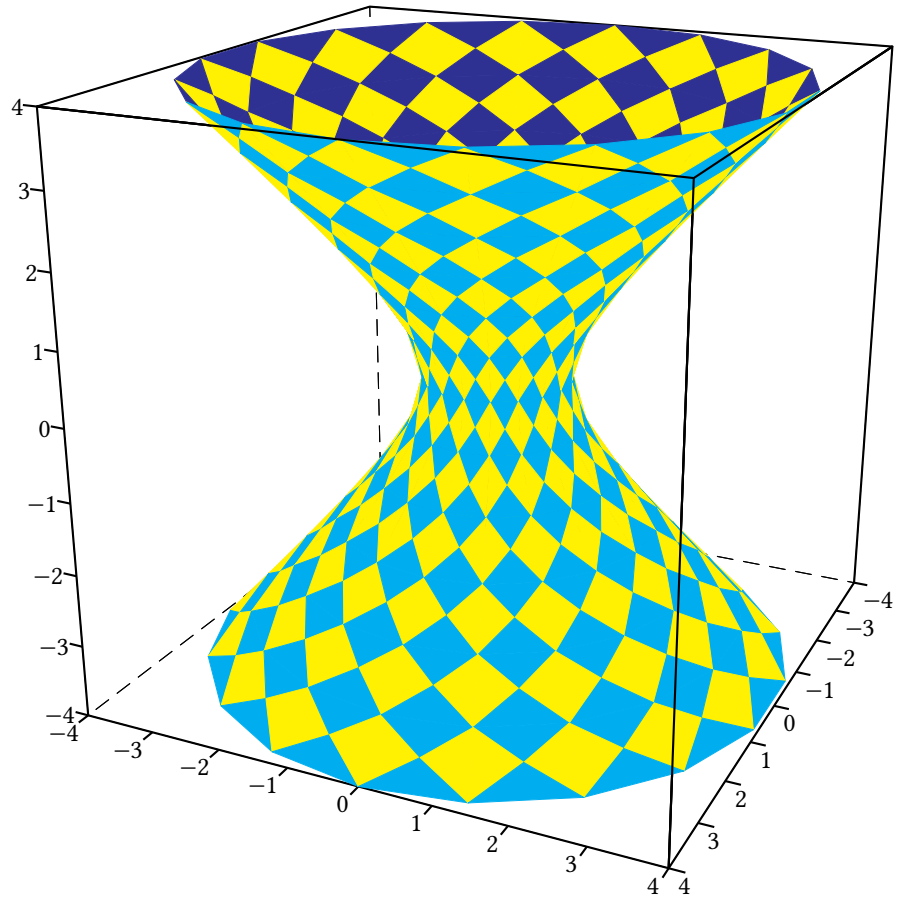
If one wishes to mesh only the exterior, the option to be used—after calculation—is:

```
fcol= n1*n2*4 2  n1*n2*4*2{
  /Rang exch def
  Rang (Blue) Rang 1 add (Yellow)
} for
```



First example

```
\begin{pspicture}(-8,-7.2)(6,6)
\psset{viewpoint=25 25 30 rtp2xyz,lightsrc=viewpoint,Decran=25}
\gridIIID[Zmin=-4,Zmax=4,showAxes=false](-4,4)(-4,4)
\psHyperboloid[
  hollow,linewidth=0.25\pslinewidth,grid,
  % inouthue=1 0 0.5 1,
  fcol= 0 2 5760{
     /Rang exch def
     Rang (Black) Rang 1 add (White)
  } for,
  ngrid=36 20]
\psLineIIID(4,-4,4)(4,4,4)(-4,4,4)
\psLineIIID(4,4,-4)(4,4,4)
\end{pspicture}
```



First example

```
\begin{pspicture}(-8,-7)(6,5)
\psset{viewpoint=25 25 20 rtp2xyz,lightsrc=,Decran=25}
\gridIIID[Zmin=-4,Zmax=4,showAxes=false](-4,4)(-4,4)
\psHyperboloid[
  hollow,linewidth=0.25\pslinewidth,grid,% inouthue=1 0 0.5 1,
  fcol= 0 2 862{/Rang exch def
    Rang (Blue) Rang 1 add (Yellow)
  } for
  864 2 1726{/Rang exch def
    Rang (Cyan) Rang 1 add (Yellow)
  } for,
  ngrid=18 12
]
\psLineIIID(4,-4,4)(4,4,4)(-4,4,4)
\psLineIIID(4,4,-4)(4,4,4)
\end{pspicture}
```

2 Hyperboloid of one sheet

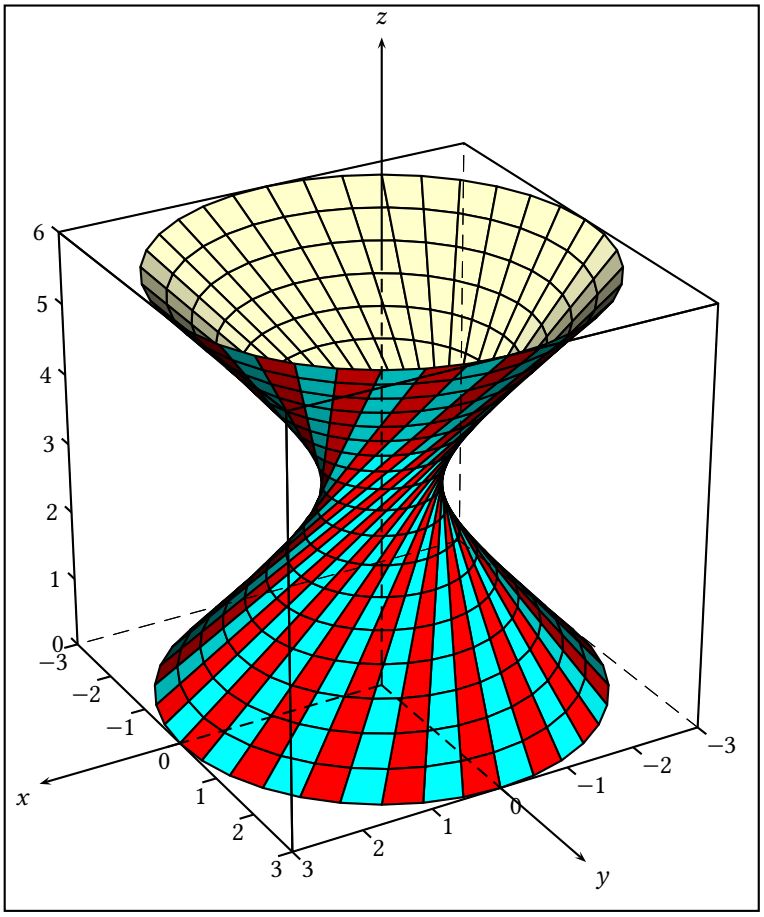
The hyperboloid of one sheet is a ruled surface. We consider the generator line $\overline{A_1A_2}$, defined by the cylinder's height H, its radius R, and the angular offset θ between the two endpoints.

From this, we derive the parametric equations for the hyperboloid:

$$\begin{cases} [l]x = & R\big((1-k)\cos(t) + k\cos(t+\theta)\big) \\ y = & R\big((1-k)\sin(t) + k\sin(t+\theta)\big) \\ z = & kH \end{cases}$$

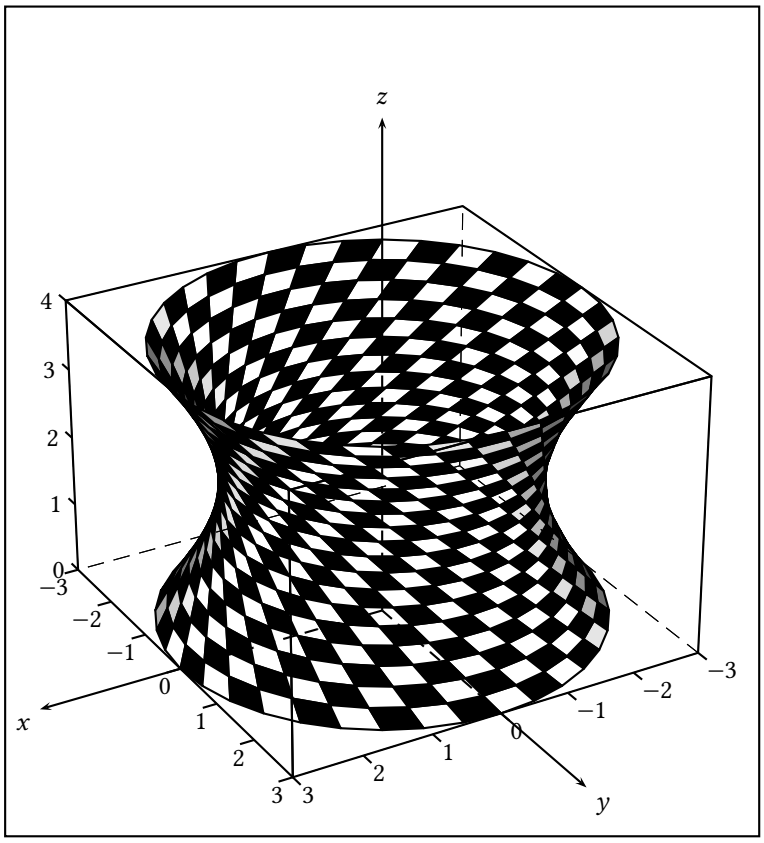
The parameter ranges are: $k \in [0, 1]$ and $t \in [0, 2\pi]$. The value of θ controls the constriction at the center of the hyperboloid. Depending on the sign of θ, the generator lines spiral to the right or to the left.

2.1 Examples and various possible representations



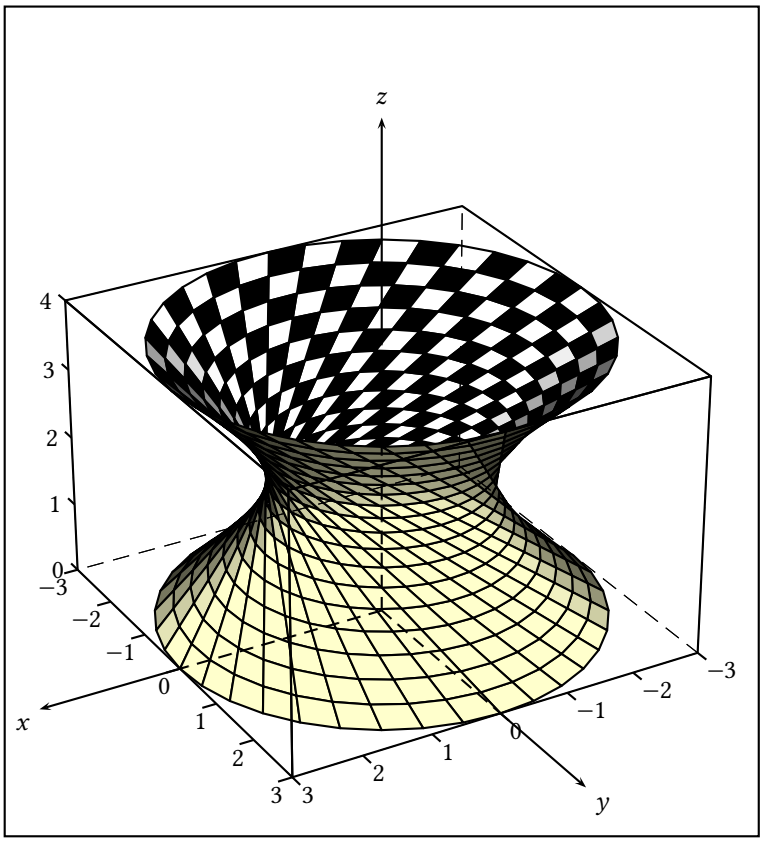
Example 1

```
\begin{pspicture}(-8,-3)(8,9)
\psset{unit=1cm}
\psset{viewpoint=50 60 30 rtp2xyz,lightsrc=viewpoint,Decran=50}
\psframe(-5,-3)(5,9)
\pstVerb{/iface 0 def
         /decalage 150 3.14159265359 mul 180 div def /H 6 def /Rayon 3 def}%
\defFunction[algebraic]{hyperboloidonesheet}(u,v)%
  {Rayon*(cos(u)*(1-v)+v*(cos(u+decalage)))}
  {Rayon*(sin(u)*(1-v)+v*(sin(u+decalage)))}
  {v*H}
\psSolid[object=surfaceparametree,
  ngrid=36 18, hue=0 1 0.5 1, %fillcolor=JaunePale,
  incolor=yellow!20,%show=all,num=all,
  fcol= 0 1 35{  % PS code for the color setting
    /Rang exch def
    Rang 2 div cvi 2 mul cvi
    Rang eq {/Damier {iface (Red)} def}{/Damier {iface (Cyan)} def} ifelse
    18 { Damier /iface iface 1 add def } repeat
  } for, % end fcol=
  base=0 2 Pi mul 0 1, function=hyperboloidonesheet]
\gridIIID[Zmin=0,Zmax=6](-3,3)(-3,3)
\end{pspicture}
```



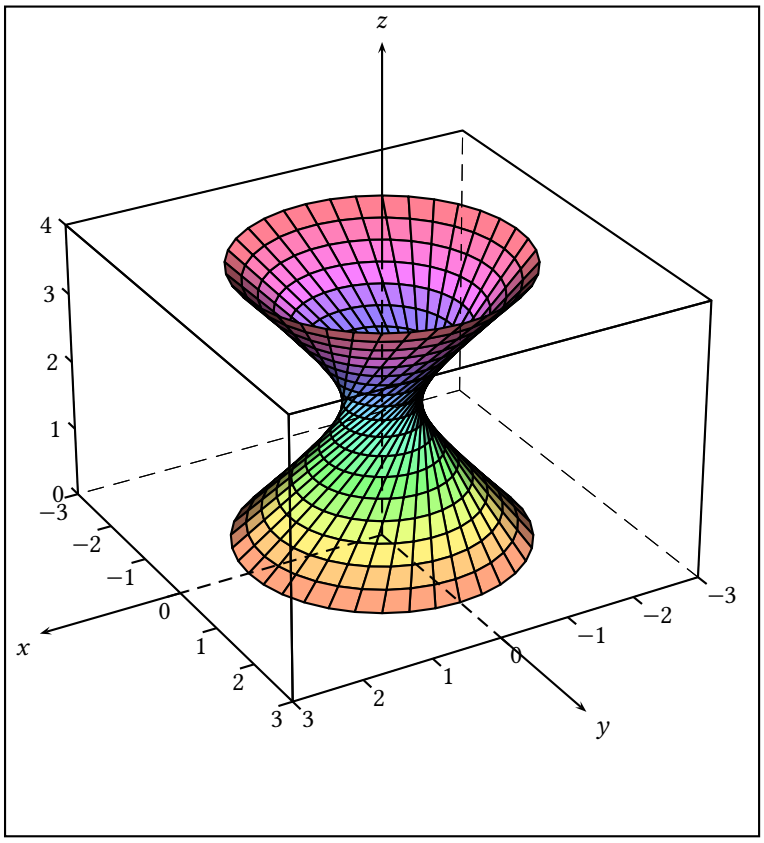
Example 2

```
\begin{pspicture}(-8,-3)(8,8)
\psset{unit=1cm}
\psset{viewpoint=50 60 30 rtp2xyz,lightsrc=viewpoint,Decran=50}
\psframe(-5,-3)(5,8)
\pstVerb{/iface 0 def
        /decalage -90 3.14159265359 mul 180 div def /H 4 def /Rayon 3 def}%
\defFunction[algebraic]{hyperboloidonesheet}(u,v)%
  {Rayon*(cos(u)*(1-v)+v*(cos(u+decalage)))}
  {Rayon*(sin(u)*(1-v)+v*(sin(u+decalage)))}
  {v*H}%
\psSolid[object=surfaceparametree, ngrid=36 18,
  fcol= 0 1 71{/Rang exch def
               Rang 2 div cvi 2 mul cvi
               Rang eq {/Damier {iface (Black) iface 1 add (White)} def}{/Damier {iface (White) iface 1 add (Black)}
                ↪  def}ifelse
    9{
       Damier
       /iface iface 2 add def
    } repeat
  } for,
  base=0 2 Pi mul 0 1, function=hyperboloidonesheet]
\gridIIID[Zmin=0,Zmax=4](-3,3)(-3,3)
\end{pspicture}
```



Example 3

```
\begin{pspicture}(-8,-3)(8,8)
\psset{unit=1cm}
\psset{viewpoint=50 60 30 rtp2xyz,lightsrc=viewpoint,Decran=50}
\psframe(-5,-3)(5,8)
\pstVerb{/iface 0 def
         /decalage -120 3.14159265359 mul 180 div def /H 4 def /Rayon 3 def}%
\defFunction[algebraic]{hyperboloidonesheet}(u,v)%
  {Rayon*(cos(u)*(1-v)+v*(cos(u+decalage)))}
  {Rayon*(sin(u)*(1-v)+v*(sin(u+decalage)))}
  {v*H}%
\psSolid[object=surfaceparametree,
  ngrid=36 18, incolor=yellow!20,
  fcol= 0 1 35{/Rang exch def
     Rang 2 div cvi 2 mul cvi Rang eq {/Damier {iface (Black) iface 1 add (White)} def}{/Damier {iface (White) iface 1
      ↪  add (Black)} def}ifelse
     9 { Damier /iface iface 2 add def } repeat
  } for, base=0 2 Pi mul 1 0, function=hyperboloidonesheet]
\gridIIID[Zmin=0,Zmax=4](-3,3)(-3,3)
\end{pspicture}
```



Example 3

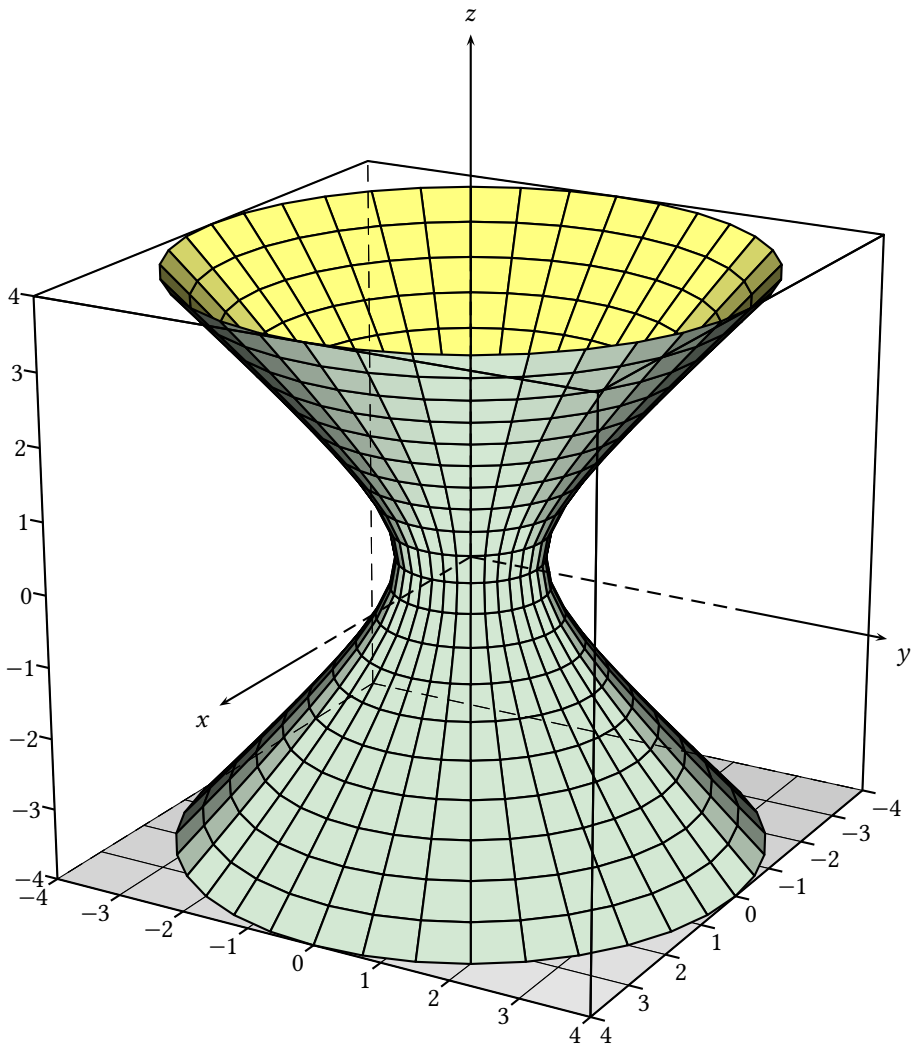
```
\begin{pspicture}(-5,-4)(5,7)
\psset{viewpoint=50 60 30 rtp2xyz,lightsrc=viewpoint,Decran=50}
\psframe(-5,-4)(5,7)
\pstVerb{/decalage 150 3.14159265359 mul 180 div def /H 4 def /Rayon 2 def}%
\defFunction[algebraic]{hyperboloidonesheet}(u,v)%
  {Rayon*(cos(u)*(1-v)+v*(cos(u+decalage)))}
  {Rayon*(sin(u)*(1-v)+v*(sin(u+decalage)))}
  {v*H}%
\psSolid[object=surfaceparametree,dotsize=3.5pt,
  ngrid=36 18,
  tablez=0 0.1 4 {} for,
 zcolor=0 1 0.5 1,
  base=0 2 Pi mul 1 0 ,
  function=hyperboloidonesheet]
\gridIIID[Zmin=0,Zmax=4](-3,3)(-3,3)
\end{pspicture}
```

3 Sections of a Hyperboloid of One Sheet

The radius of the generating hyperbola varies from bottom to top, following the relation:

$$\frac{r^2}{a^2} = 4k^2\left(\frac{R^2}{a^2} - 1\right) + 1$$

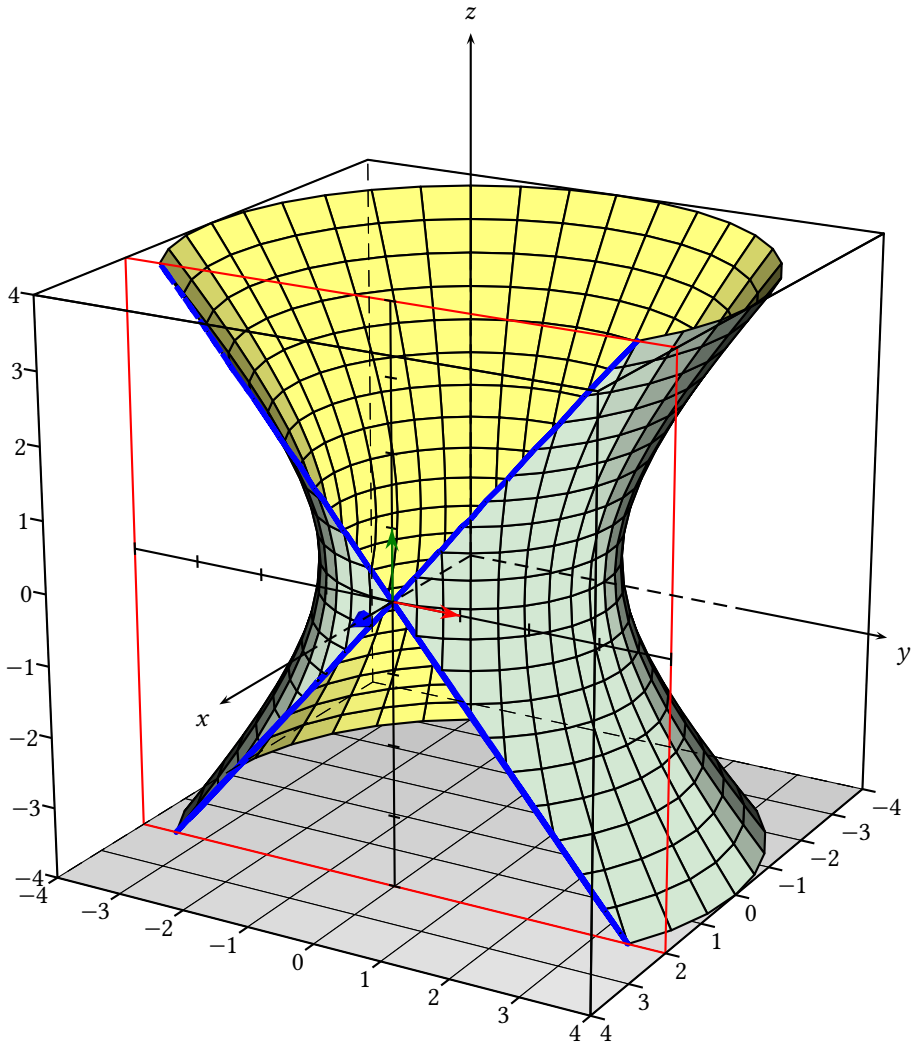
$$r = \sqrt{4k^2(R^2 - a^2) + a^2} \text{ with } -\frac{1}{2} \leqslant k \leqslant \frac{1}{2}$$



Example 4

```
\begin{pspicture}(-8,-7)(8,7)
\psset{solidmemory, viewpoint=50 30 20 rtp2xyz,lightsrc=viewpoint,Decran=50}
\pstVerb{/Hauteur 8 def
         /nEtages 20 def
         /Pas 1 nEtages div def
         /nMeridiens 36 def
         /DemiAxeFocal 1 def % rayon de l'étranglement au milieu
         /RayonBases 4 def}%
\psSolid[object=grille,base=-4 4 -4 4,linewidth=0.5\pslinewidth](0,0,Hauteur 2 div neg)
\psSolid[object=new, incolor=yellow!50, fillcolor=ForestGreen!20, hollow,
         sommets=0.5 Pas neg -0.5 Pas neg add {
           /k exch def
           0 1 nMeridiens 1 sub {
```

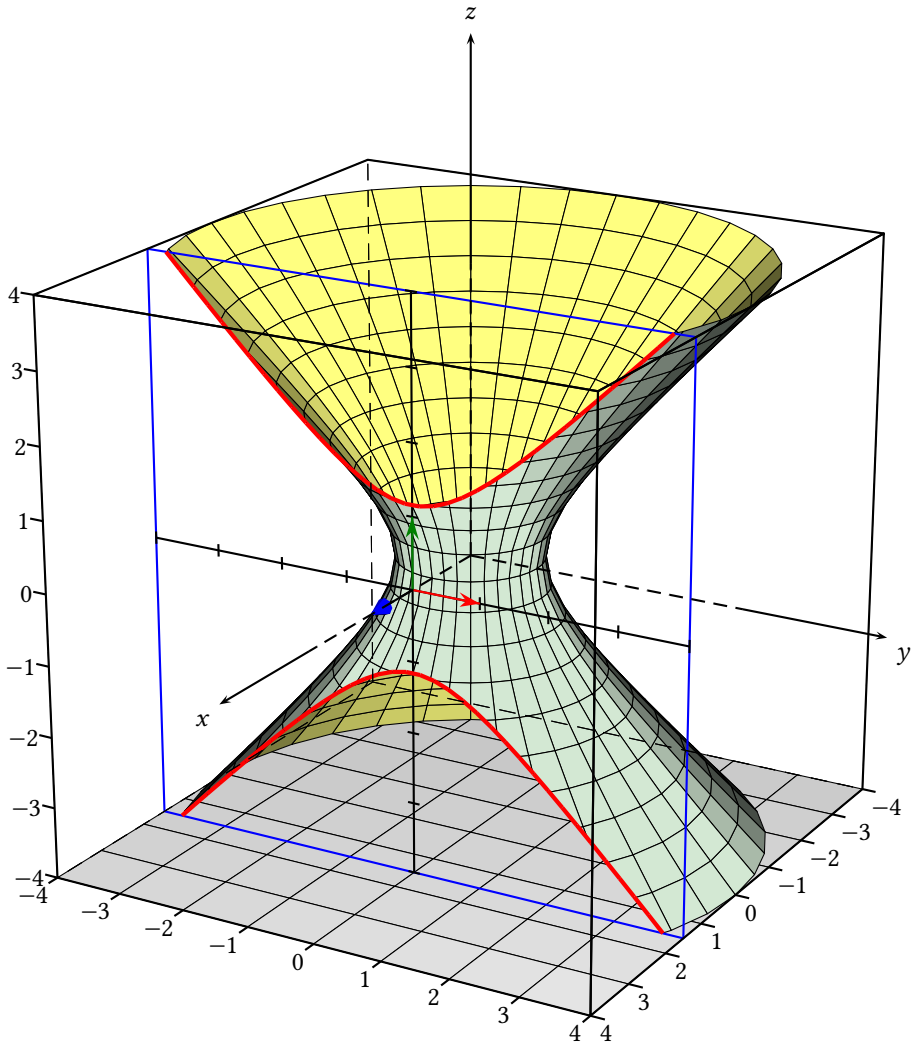
```
            /i exch def
            /r 4 k dup mul mul RayonBases dup mul DemiAxeFocal dup mul sub mul
                 DemiAxeFocal dup mul add sqrt def
            360 nMeridiens div i mul cos r mul
            360 nMeridiens div i mul sin r mul
            k Hauteur mul
          } for
        } for,
      faces={ 0 1 nEtages 1 sub {
        /k1 exch def
        k1 nMeridiens mul 1 add 1 k1 1 add nMeridiens mul 1 sub {
          /i exch def
          [i i 1 sub nMeridiens i add 1 sub nMeridiens i add]
        } for
        [k1 nMeridiens mul k1 1 add nMeridiens mul 1 sub k1 2 add nMeridiens mul 1 sub k1 1 add nMeridiens mul]
      } for
    }]% end \pssolid
\gridIIID[Zmin=-4,Zmax=4](-4,4)(-4,4)
\end{pspicture}
```



Example 5

```
\begin{pspicture}(-8,-7)(8,7)
\psset{solidmemory,viewpoint=50 30 20 rtp2xyz,lightsrc=viewpoint,Decran=50}
\pstVerb{/Hauteur 8 def
         /nEtages 20 def
         /Pas 1 nEtages div def
         /nMeridiens 36 def
         /DemiAxeFocal 2 def % rayon de l'étranglement au milieu
         /RayonBases 4 def}%
\psSolid[object=grille,base=-4 4 -4 4,linewidth=0.5\pslinewidth](0,0,Hauteur 2 div neg)
\psSolid[object=new,fillcolor=red!50,incolor=yellow,hollow,action=draw**,
  sommets=0.5 Pas neg -0.5 Pas neg add{
    /k exch def
    0 1 nMeridiens 1 sub {
      /i exch def
      /r 4 k dup mul mul RayonBases dup mul DemiAxeFocal dup mul sub mul DemiAxeFocal dup mul add sqrt def
      360 nMeridiens div i mul cos r mul
      360 nMeridiens div i mul sin r mul
      k Hauteur mul
    } for
  } for,
  faces={
    0 1 nEtages 1 sub {
      /k1 exch def
      k1 nMeridiens mul 1 add 1 k1 1 add nMeridiens mul 1 sub {
```

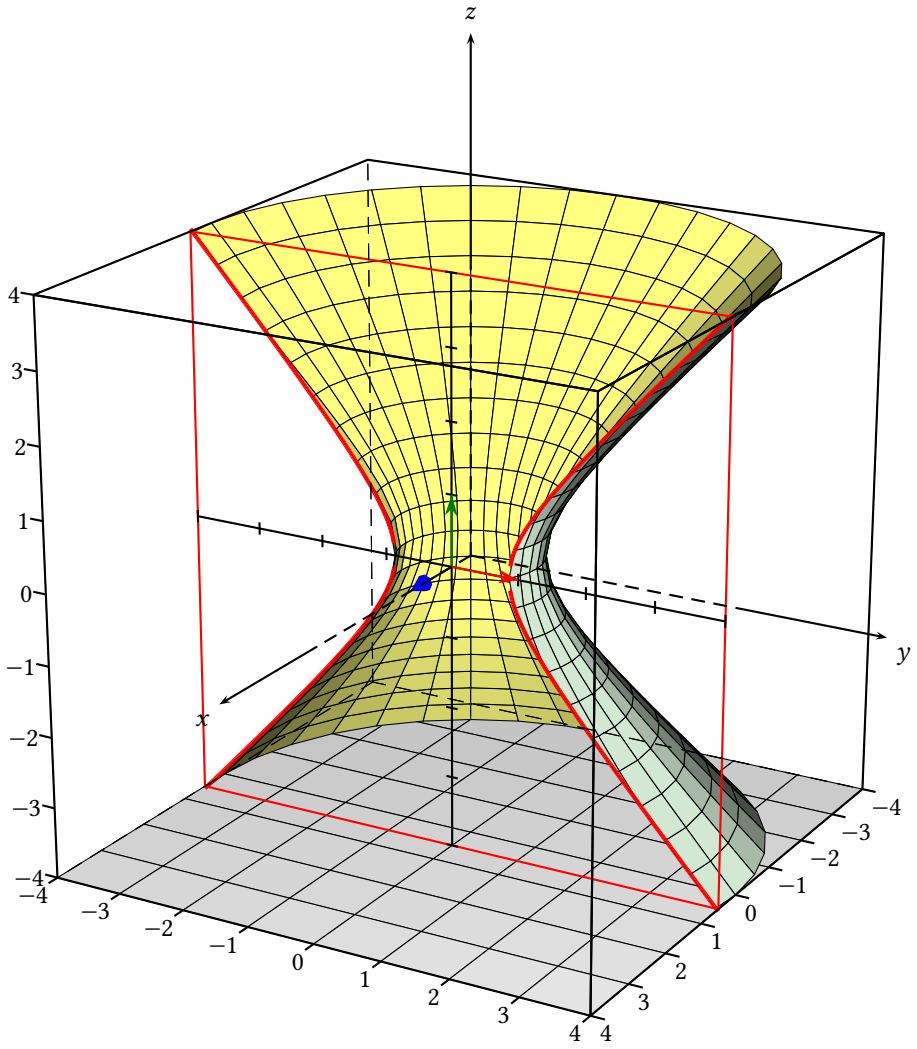
```
      /i exch def
      [i i 1 sub nMeridiens i add 1 sub nMeridiens i add]
    } for
    [k1 nMeridiens mul k1 1 add nMeridiens mul 1 sub k1 2 add nMeridiens mul 1 sub k1 1 add nMeridiens mul]
  } for
 },
 plansepare={[1 0 0 DemiAxeFocal neg]}, name=coupeHyperboloidTest,action=none]
\psSolid[object=load,incolor=yellow!50,fillcolor=ForestGreen!20,rm=2 609,
  intersectionplan={[1 0 0 DemiAxeFocal neg]},intersectioncolor=(bleu), intersectionlinewidth=2,
  intersectiontype=0, load=coupeHyperboloidTest1]
\psSolid[object=plan, definition=equation,args={[1 0 0 DemiAxeFocal neg] 90},linecolor=red,
%   fillcolor=Aquamarine,
  planmarks,base=-4 4 -4 4,showBase,action=draw]
\gridIIID[Zmin=-4,Zmax=4](-4,4)(-4,4)
\end{pspicture}
```



Example 6

```
\begin{pspicture}(-8,-7)(8,7)
\psset{solidmemory, viewpoint=50 30 20 rtp2xyz,lightsrc=viewpoint,Decran=50}
\pstVerb{/Hauteur 8 def          /nEtages 20 def
         /Pas 1 nEtages div def  /nMeridiens 36 def
         /DemiAxeFocal 1 def % rayon de l'étranglement au milieu
         /RayonBases 4 def
         /DemiAxeNonFocal Hauteur 2 div RayonBases dup mul DemiAxeFocal dup mul sub sqrt div DemiAxeFocal mul def
         /X0 1.5 def    /Z0 DemiAxeNonFocal X0 DemiAxeFocal div dup mul 1 sub sqrt mul def
         /Ymax DemiAxeFocal Hauteur 2 DemiAxeNonFocal mul div dup mul 1 add X0 DemiAxeFocal div dup mul sub sqrt mul
         ↪  def}%
\psSolid[object=grille,base=-4 4 -4 4,linewidth=0.5\pslinewidth](0,0,Hauteur 2 div neg)
\psSolid[object=new,incolor=yellow!50, fillcolor=ForestGreen!20, hollow,
         sommets=
           0.5 Pas neg -0.5 Pas neg add{
             /k exch def
             0 1 nMeridiens 1 sub {
               /i exch def
               /r 4 k dup mul mul RayonBases dup mul DemiAxeFocal dup mul sub mul DemiAxeFocal dup mul add sqrt def
               360 nMeridiens div i mul cos r mul
               360 nMeridiens div i mul sin r mul
               k Hauteur mul
             } for
           } for,
    faces={ 0 1 nEtages 1 sub {
```

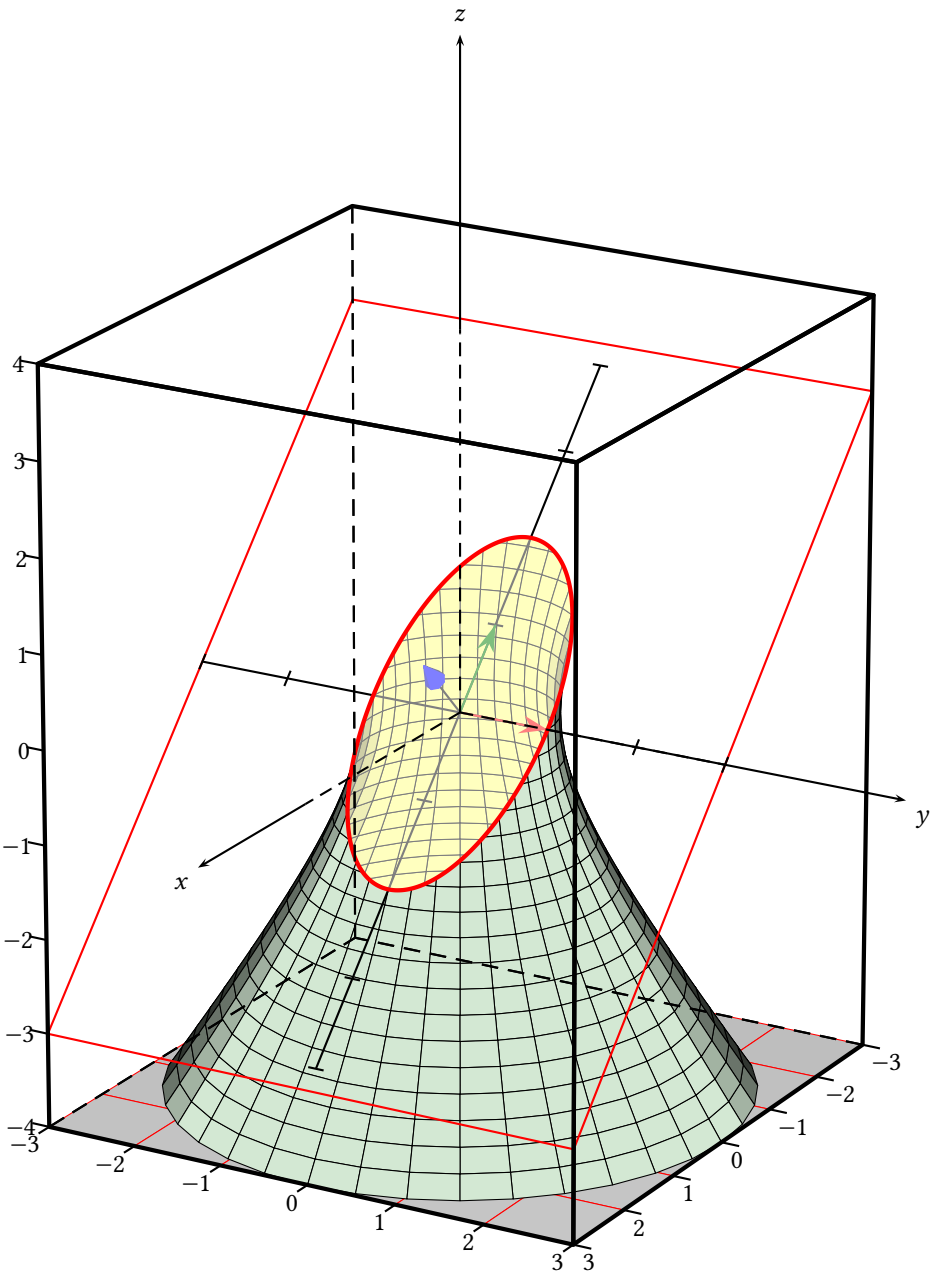
```
         /k1 exch def
           k1 nMeridiens mul 1 add 1 k1 1 add nMeridiens mul 1 sub {
             /i exch def
             [i i 1 sub nMeridiens i add 1 sub nMeridiens i add]
           } for
           [k1 nMeridiens mul k1 1 add nMeridiens mul 1 sub k1 2 add nMeridiens mul 1 sub k1 1 add nMeridiens mul]
         } for},
  plansepare={[1 0 0 -1.5]},name=coupeHyperboloidTest,action=none]
\psSolid[object=load,deactivatecolor=true, %fillcolor=red!50,incolor=yellow,rm=0,
        linewidth=0.5\pslinewidth,rm=0,load=coupeHyperboloidTest1]
\psSolid[object=plan,definition=equation,args={[1 0 0 -1.5] 90},linecolor=blue,%   fillcolor=Aquamarine,
   planmarks,base=-4 4 -4 4,name=PlanCoupe,showBase,action=draw]
\defFunction[algebraic]{hyperbol1}(t){X0}{t}{sqrt(((X0^2+t^2)/(DemiAxeFocal^2)-1))*DemiAxeNonFocal}
\defFunction[algebraic]{hyperbol2}(t){X0}{t}{-sqrt(((X0^2+t^2)/(DemiAxeFocal^2)-1))*DemiAxeNonFocal}
\psSolid[object=courbe,linewidth=2\pslinewidth,function=hyperbol1,range=Ymax neg Ymax,r=0,linecolor=red]%
\psSolid[object=courbe,linewidth=2\pslinewidth,function=hyperbol2,range=Ymax neg Ymax,r=0,linecolor=red]%
\gridIIID[Zmin=-4,Zmax=4](-4,4)(-4,4)
\end{pspicture}
```



Example 6

```
\begin{pspicture}(-8,-7)(8,7)
\psset{solidmemory,viewpoint=50 30 20 rtp2xyz,lightsrc=viewpoint,Decran=50,resolution=200}
\pstVerb{/Hauteur 8 def         /nEtages 20 def
         /Pas 1 nEtages div def /nMeridiens 36 def
         /DemiAxeFocal 1 def % rayon de l'étranglement au milieu
         /RayonBases 4 def
         /DemiAxeNonFocal Hauteur 2 div RayonBases dup mul DemiAxeFocal dup mul sub sqrt div DemiAxeFocal mul def
         /X0 0.5 def  /Y0 DemiAxeFocal 1 X0 DemiAxeFocal div dup mul sub sqrt mul def
         /Ymax RayonBases def}%
\psSolid[object=grille,base=-4 4 -4 4,linewidth=0.5\pslinewidth](0,0,Hauteur 2 div neg)%
\psSolid[object=new,fillcolor=red!50,incolor=yellow,hollow,action=draw**,
  sommets=
    0.5 Pas neg -0.5 Pas neg add{
    /k exch def
    0 1 nMeridiens 1 sub {
        /i exch def
        /r 4 k dup mul mul RayonBases dup mul DemiAxeFocal dup mul sub mul DemiAxeFocal dup mul add sqrt def
        360 nMeridiens div i mul cos r mul
        360 nMeridiens div i mul sin r mul
        k Hauteur mul
      } for
    } for
    0 1 nMeridiens 1 sub {/J exch def
        RayonBases 360 nMeridiens div J mul cos mul
```

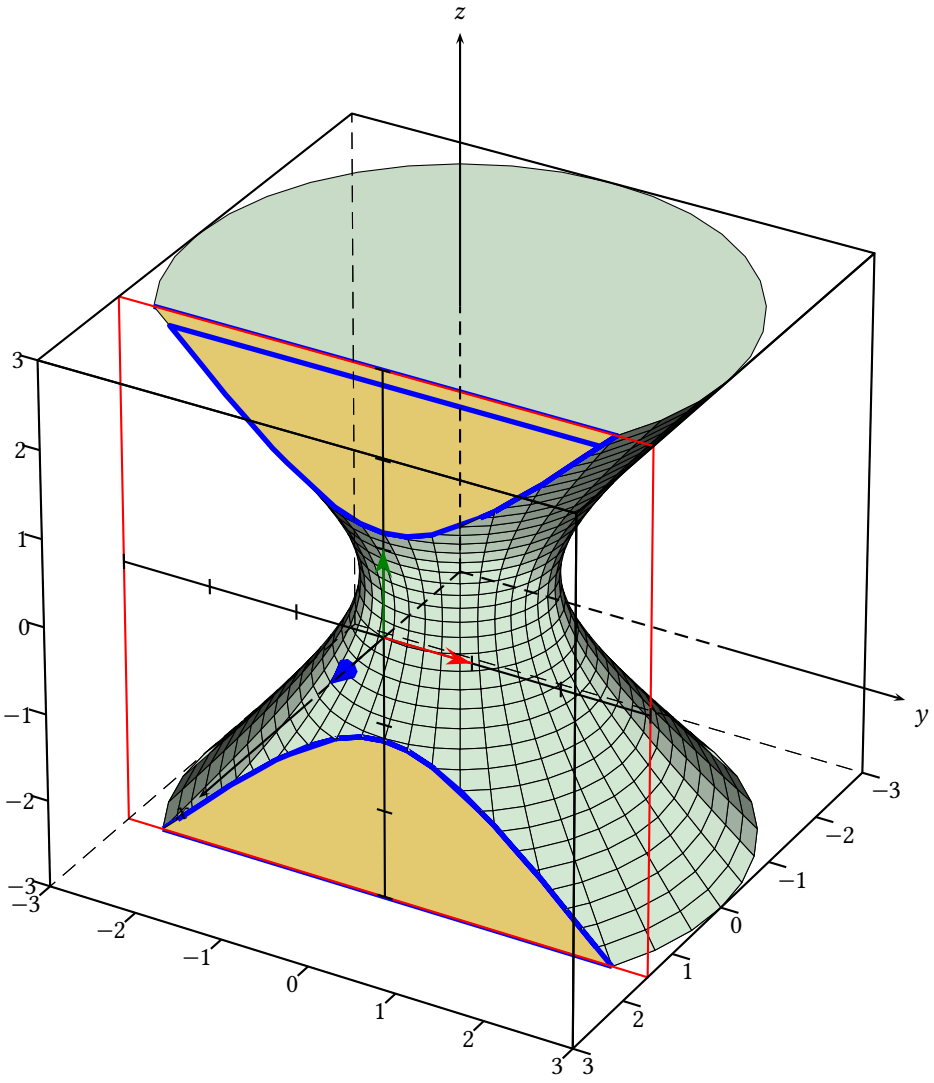
```
       RayonBases 360 nMeridiens div J mul sin mul
     Hauteur 2 div } for,
  faces={
    0 1 nEtages 1 sub {
    /k1 exch def
    k1 nMeridiens mul 1 add 1 k1 1 add nMeridiens mul 1 sub {
      /i exch def
      [i i 1 sub nMeridiens i add 1 sub nMeridiens i add]
    } for
    [k1 nMeridiens mul k1 1 add nMeridiens mul 1 sub k1 2 add nMeridiens mul 1 sub k1 1 add nMeridiens mul]
    } for},
  plansepare={[1 0 0 -0.5]},name=coupeHyperboloidTest, action=none]
\psSolid[object=load,incolor=yellow!50,fillcolor=ForestGreen!20,linewidth=0.5\pslinewidth,
        rm=0 1,%numfaces=all,%rm=0,
        % intersectionplan={[1 0 0 -0.5]},
        % intersectioncolor=(bleu),
        % intersectionlinewidth=2,
        % intersectiontype=0,
        load=coupeHyperboloidTest1]
\defFunction[algebraic]{hyperbol3}(t){X0}{t}{DemiAxeNonFocal*sqrt((t^2+X0^2)/(DemiAxeFocal^2)-1)}
\defFunction[algebraic]{hyperbol4}(t){X0}{t}{-DemiAxeNonFocal*sqrt((t^2+X0^2)/(DemiAxeFocal^2)-1)}
\psSolid[object=courbe,linewidth=2\pslinewidth,function=hyperbol3,range=Y0 neg -4,
   r=0,linecolor=red]%
\psSolid[object=courbe,linewidth=2\pslinewidth,
   function=hyperbol3,range=Y0 4,r=0,linecolor=red]%
\psSolid[object=courbe,linewidth=2\pslinewidth,
   function=hyperbol4,range=Y0 4,r=0,linecolor=red]%
\psSolid[object=courbe,linewidth=2\pslinewidth,
   function=hyperbol4,range=Y0 neg -4,r=0,linecolor=red]%
\psSolid[object=plan, definition=equation,args={[1 0 0 -0.5] 90},linecolor=red,
   planmarks, base=-4 4 -4 4, showBase,action=draw
]
\gridIIID[Zmin=-4,Zmax=4](-4,4)(-4,4)
%\psPoint(X0,Y0,0){P}\psdot(P)
\end{pspicture}
```



Example 7

```
\begin{pspicture}(-8,-7)(8,9)
\psset{solidmemory,viewpoint=100 30 20 rtp2xyz,lightsrc=viewpoint,Decran=133}
\pstVerb{/Hauteur 8 def          /nEtages 36 def
         /Pas 1 nEtages div def  /nMeridiens 36 def
         /DemiAxeFocal 1 def % rayon de l'étranglement au milieu
         /RayonBases 3 def
         /DemiAxeNonFocal Hauteur 2 div RayonBases dup mul DemiAxeFocal dup mul sub sqrt div DemiAxeFocal mul def
         /xMax DemiAxeNonFocal DemiAxeFocal mul DemiAxeNonFocal dup mul DemiAxeFocal dup mul sub sqrt div def
         /GrandAxe xMax 2 sqrt mul def}%
\psSolid[object=grille,base=-3 3 -3 3,linewidth=0.5\pslinewidth,linecolor=red](0,0,Hauteur 2 div neg)%
\psSolid[object=new,fillcolor=red!50,incolor=yellow,%hollow,
  action=draw*,
  sommets=
  0.5 Pas neg -0.5 Pas neg add{
    /k exch def
    0 1 nMeridiens 1 sub {
```

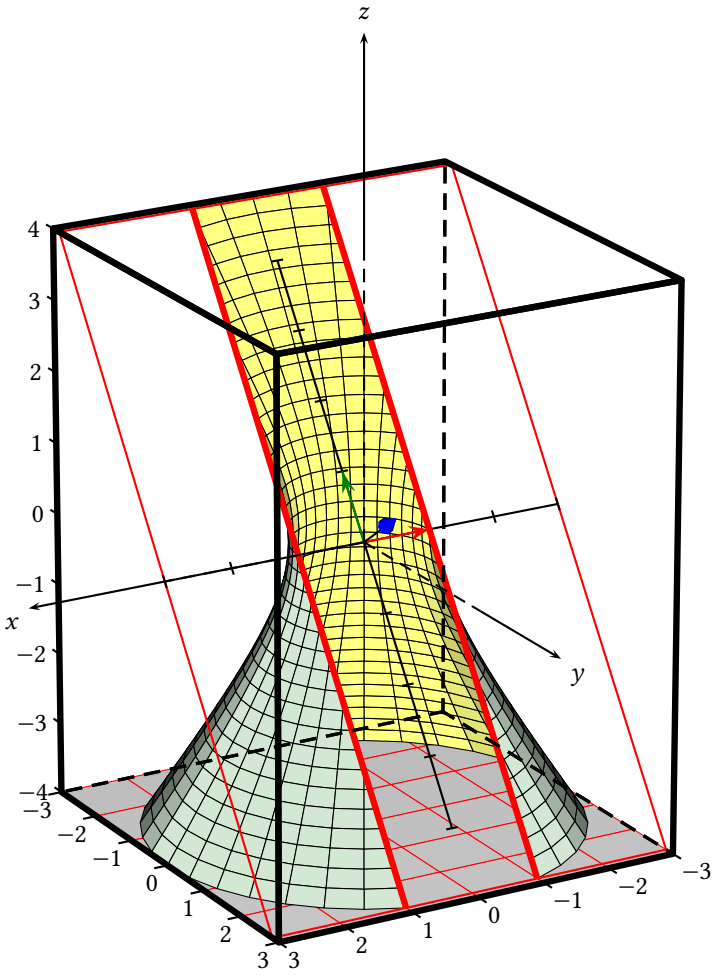
```
      /i exch def
      /r 4 k dup mul mul RayonBases dup mul DemiAxeFocal dup mul sub mul DemiAxeFocal dup mul add sqrt def
      360 nMeridiens idiv i mul cos r mul
      360 nMeridiens idiv i mul sin r mul
      k Hauteur mul
    } for
  } for,
  faces={
  0 1 nEtages 1 sub {
    /k1 exch def
    k1 nMeridiens mul 1 add 1 k1 1 add nMeridiens mul 1 sub {
      /i exch def
      [i i 1 sub nMeridiens i add 1 sub nMeridiens i add]
    } for
    [k1 nMeridiens mul k1 1 add nMeridiens mul 1 sub k1 2 add nMeridiens mul 1 sub k1 1 add nMeridiens mul]
    } for
  % faces du dessus et de dessous
  %[0 1 nMeridiens 1 sub {}for]
  %[nMeridiens nEtages 1 add mul 1 sub -1  nMeridiens nEtages mul {}for ]
  },plansepare={[1 0 1 0]}, name=coupeHyperboloidTest,action=none]
\psSolid[object=load,incolor=yellow!50, fillcolor=ForestGreen!20,linewidth=0.5\pslinewidth,
   rm=0,load=coupeHyperboloidTest1,hollow=true](0,0,0)
\defFunction[algebraic]{ellipse}(t){DemiAxeFocal*cos(t)}{GrandAxe*sin(t)}{}
\psSolid[object=plan,definition=equation,args={[1 0 1 0] 90},linecolor=red,fillcolor=Aquamarine,
   planmarks, base=-3 3 -3 2 sqrt mul 3 2 sqrt mul,showBase,action=draw
]%
\defFunction[algebraic]{ellipse}(t){DemiAxeFocal*cos(t)}{GrandAxe*sin(t)}{}
\psSolid[object=plan,definition=equation,
   args={[1 0 1 0] 90},base=-10 10 -10 10,action=none,name=monplan]%
\psProjection[object=courbeR2,plan=monplan,range=0 2 pi mul,resolution=360,
   linecolor=red,fillstyle=solid,opacity=0.5,linewidth=2\pslinewidth,function=ellipse
]%
\composeSolid
%\psPoint(0,DemiAxeFocal,0){F}\psdot(F)
%\psPoint(xMax neg,0,xMax){F2}\psdot[linecolor=red](F2)
\gridIIID[Zmin=-4,Zmax=4,linewidth=1\pslinewidth](-3,3)(-3,3)
\end{pspicture}
```



Example 8

```
\begin{pspicture}(-8,-7)(8,7)
\psset{solidmemory,viewpoint=100 30 30 rtp2xyz,lightsrc=viewpoint,Decran=133}
\pstVerb{/Hauteur 6 def          /nEtages 36 def
         /Pas 1 nEtages div def  /nMeridiens 36 def
         /DemiAxeFocal 1 def % rayon de l'étranglement au milieu
         /RayonBases 3 def       /JaunePale {0.89 0.79 0.435 setrgbcolor} def}%
\psSolid[object=new,fillcolor=red!50,incolor=yellow,%hollow,
  action=draw*,sommets=
  0.5 Pas neg -0.5 Pas neg add{
    /k exch def
    0 1 nMeridiens 1 sub {
      /i exch def
      /r 4 k dup mul mul RayonBases dup mul DemiAxeFocal dup mul sub mul DemiAxeFocal dup mul add sqrt def
      360 nMeridiens idiv i mul cos r mul
      360 nMeridiens idiv i mul sin r mul
      k Hauteur mul
    } for
  } for,
  faces={
  %face du dessus
  [0 1 nMeridiens 1 sub {}for]
  % les faces latérales
  0 1 nEtages 1 sub {
```

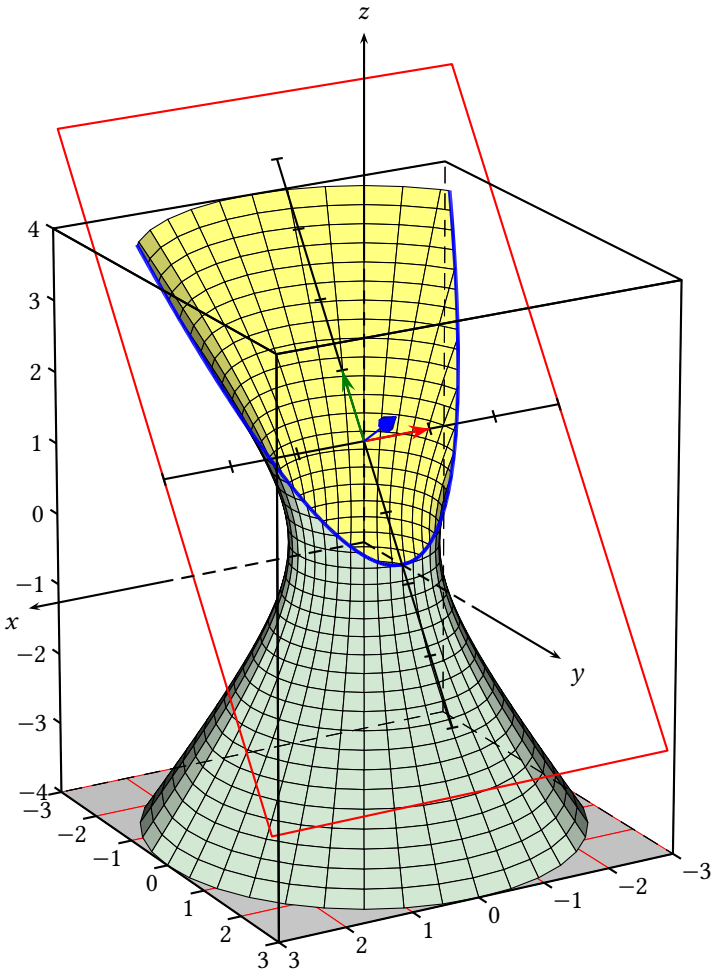
```
    /k1 exch def
    k1 nMeridiens mul 1 add 1 k1 1 add nMeridiens mul 1 sub {
    /i exch def
    [i i 1 sub nMeridiens i add 1 sub nMeridiens i add]
    } for
    [k1 nMeridiens mul k1 1 add nMeridiens mul 1 sub k1 2 add nMeridiens mul 1 sub k1 1 add nMeridiens mul]
  } for
  % face de dessous
  %[nMeridiens nEtages 1 add mul 1 sub -1  nMeridiens nEtages mul {}for ]
  [nMeridiens nEtages mul 1 nMeridiens nEtages 1 add mul 1 sub {}for ]
  },plansepare={[1 0 0 -1.5]},name=coupeHyperboloidTest,action=none]
  %   file=hyperboloideVersion2,
  %   action=writesolid]
  %\psSolid[object=datfile,
  %         file=hyperboloideVersion2, fillcolor=red!50,incolor=yellow, linewidth=0.5\pslinewidth
  %         ]
\psSolid[object=load,incolor=yellow!50,fillcolor=ForestGreen!20, fcol=0 (JaunePale) 1 (JaunePale),% 1078 (JaunePale),
  linewidth=0.5\pslinewidth, % numfaces=all,
  intersectionplan={[1 0 0 -1.5]},intersectioncolor=(bleu),intersectionlinewidth=2,
  intersectiontype=0,load=coupeHyperboloidTest1,hollow=false]
\psSolid[object=plan,definition=equation,args={[1 0 0 -1.5] 90},linecolor=red,
   fillcolor=Aquamarine,planmarks,base=-3 3 -3 3,showBase,action=draw]
\gridIIID[Zmin=-3,Zmax=3](-3,3)(-3,3)
\end{pspicture}
```



Example 8

```
\begin{pspicture}(-8,-7)(8,7)
\psset{solidmemory,viewpoint=100 60 20 rtp2xyz,lightsrc=viewpoint,Decran=100}
\pstVerb{/Hauteur 8 def           /nEtages 36 def
         /Pas 1 nEtages div def   /nMeridiens 36 def
         /DemiAxeFocal 1 def % rayon de l'étranglement au milieu
         /RayonBases 3 def
         /DemiAxeNonFocal Hauteur 2 div RayonBases dup mul DemiAxeFocal dup mul sub sqrt div DemiAxeFocal mul def
         /xMax DemiAxeNonFocal DemiAxeFocal mul DemiAxeNonFocal dup mul DemiAxeFocal dup mul sub sqrt div def
         /GrandAxe xMax 2 sqrt mul def   /ConeAsymptote DemiAxeFocal DemiAxeNonFocal div def
}%
\psSolid[object=grille,base=-3 3 -3 3,linewidth=0.5\pslinewidth,linecolor=red](0,0,Hauteur 2 div neg)%
\psSolid[object=new,fillcolor=red!50,incolor=yellow,%hollow,
  action=draw*,sommets=
    0.5 Pas neg -0.5 Pas neg add{
      /k exch def
    0 1 nMeridiens 1 sub {
      /i exch def
      /r 4 k dup mul mul RayonBases dup mul DemiAxeFocal dup mul sub mul DemiAxeFocal dup mul add sqrt def
      360 nMeridiens idiv i mul cos r mul
      360 nMeridiens idiv i mul sin r mul
      k Hauteur mul
    } for
  } for, faces={
  0 1 nEtages 1 sub {
    /k1 exch def
    k1 nMeridiens mul 1 add 1 k1 1 add nMeridiens mul 1 sub {
    /i exch def
```

```
    [i i 1 sub nMeridiens i add 1 sub nMeridiens i add]
    } for
    [k1 nMeridiens mul k1 1 add nMeridiens mul 1 sub k1 2 add nMeridiens mul 1 sub k1 1 add nMeridiens mul]
  } for
  % faces du dessus et de dessous
  %[0 1 nMeridiens 1 sub {}for]
  %[nMeridiens nEtages 1 add mul 1 sub -1  nMeridiens nEtages mul {}for ]
  }, plansepare={[0 1 ConeAsymptote 0]},name=coupeHyperboloidTest,action=none]
\psSolid[object=load,incolor=yellow!50,fillcolor=ForestGreen!20, linewidth=0.5\pslinewidth,
  rm=0,% intersectionplan={[0 1 ConeAsymptote 0]}, intersectioncolor=(bleu),
  %         intersectionlinewidth=2,intersectiontype=0,
          load=coupeHyperboloidTest1,hollow=true](0,0,0)
\defFunction[algebraic]{ellipse}(t){DemiAxeFocal*cos(t)}{GrandAxe*sin(t)}{}
\psSolid[object=plan,definition=equation,args={[0 1 ConeAsymptote 0] 180},linecolor=red,
   fillcolor=Aquamarine,planmarks,base=-3 3 -4.9 4.9,showBase,action=draw]%
\psSolid[object=plan, definition=equation,args={[0 1 ConeAsymptote 0] 180},base=-5 5 -4.9 4.9,
   action=none,name=monplan]%
\psProjection[object=droite,plan=monplan,linewidth=3\pslinewidth,linecolor=red,args=1 5 1 -5]
\psProjection[object=droite,linewidth=3\pslinewidth,plan=monplan,linecolor=red,args=-1 5 -1 -5]
\composeSolid
%\psPoint(0,DemiAxeFocal,0){F}\psdot(F)
%\psPoint(xMax neg,0,xMax){F2}\psdot[linecolor=red](F2)
\gridIIID[Zmin=-4,Zmax=4,linewidth=1\pslinewidth](-3,3)(-3,3)
\end{pspicture}
```



Example 9

```
\begin{pspicture}(-8,-7)(8,7)
\psset{solidmemory}
\psset{viewpoint=100 60 20 rtp2xyz,lightsrc=viewpoint,Decran=100}
\pstVerb{
  /Hauteur 8 def  /nEtages 36 def  /Pas 1 nEtages div def  /nMeridiens 36 def
  /DemiAxeFocal 1 def % rayon de l'étranglement au milieu
  /RayonBases 3 def
  /DemiAxeNonFocal Hauteur 2 div RayonBases dup mul DemiAxeFocal dup mul sub sqrt div DemiAxeFocal mul def
  /xMax DemiAxeNonFocal DemiAxeFocal mul DemiAxeNonFocal dup mul DemiAxeFocal dup mul sub sqrt div def
  /GrandAxe xMax 2 sqrt mul def  /ConeAsymptote DemiAxeFocal DemiAxeNonFocal div def
}%
\psSolid[object=grille,base=-3 3 -3 3,linewidth=0.5\pslinewidth,linecolor=red](0,0,Hauteur 2 div neg)%
\psSolid[object=new,fillcolor=red!50,incolor=yellow,%hollow,
        action=draw*, sommets=
   0.5 Pas neg -0.5 Pas neg add{
   /k exch def
   0 1 nMeridiens 1 sub {
     /i exch def
     /r 4 k dup mul mul RayonBases dup mul DemiAxeFocal dup mul sub mul DemiAxeFocal dup mul add sqrt def
     360 nMeridiens idiv i mul cos r mul
     360 nMeridiens idiv i mul sin r mul
     k Hauteur mul
   } for
   } for,
  faces={
    0 1 nEtages 1 sub {
      /k1 exch def
```

```
      k1 nMeridiens mul 1 add 1 k1 1 add nMeridiens mul 1 sub {
        /i exch def
        [i i 1 sub nMeridiens i add 1 sub nMeridiens i add]
      } for
      [k1 nMeridiens mul k1 1 add nMeridiens mul 1 sub k1 2 add nMeridiens mul 1 sub k1 1 add nMeridiens mul]
    } for
   % faces du dessus et de dessous
   %[0 1 nMeridiens 1 sub {}for]
   %[nMeridiens nEtages 1 add mul 1 sub -1  nMeridiens nEtages mul {}for ]
  },
  plansepare={[0 1 ConeAsymptote -1]}, name=coupeHyperboloidTest, action=none]
\psSolid[object=load, incolor=yellow!50, fillcolor=ForestGreen!20, linewidth=0.5\pslinewidth,
        rm=0, load=coupeHyperboloidTest1,hollow=true](0,0,0)
\defFunction[algebraic]{parabole1}(t)
  {DemiAxeFocal*sqrt(1+(t/DemiAxeNonFocal)^2-((1-ConeAsymptote*t)/DemiAxeFocal)^2)}{1-ConeAsymptote*t}{t}
\defFunction[algebraic]{parabole2}(t)
  {-DemiAxeFocal*sqrt(1+(t/DemiAxeNonFocal)^2-((1-ConeAsymptote*t)/DemiAxeFocal)^2)}{1-ConeAsymptote*t}{t}
\psSolid[object=plan, definition=equation, args={[0 1 ConeAsymptote -1] 180},
   linecolor=red, fillcolor=Aquamarine, planmarks, base=-3 3 -4.9 4.9, showBase,action=draw
]%
\psSolid[object=courbe, r=0, range=0 4, linecolor=blue,linewidth=0.05, resolution=360, function=parabole1]%
\psSolid[object=courbe, r=0, range=0 4, linecolor=blue,linewidth=0.05, resolution=360, function=parabole2]%
%\psPoint(0,DemiAxeFocal,0){F}\psdot(F)
%\psPoint(xMax neg,0,xMax){F2}\psdot[linecolor=red](F2)
\gridIIID[Zmin=-4,Zmax=4,linewidth=1\pslinewidth](-3,3)(-3,3)
\end{pspicture}
```

4 Sections of a Hyperboloid of One Sheet

In this part the mesh of the hyperboloid is defined as a ruled surface. A dedicated macro is used to draw the hyperboloid: `\psHyperboloid` `Options` (x,y,z) The available options, along with their default values, are listed below:

R=4 : base radius;

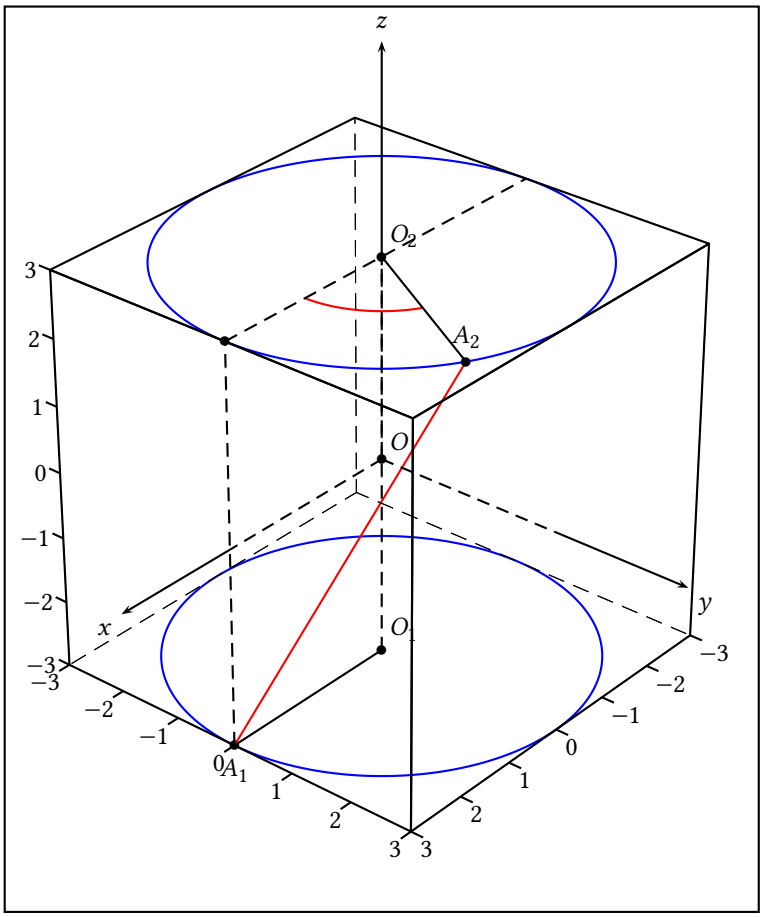
h=8 : height;

ngrid=36 20 : defines the mesh—specifically, the number of lines and the number of circles;

base=0 360 0 1 : specifies the range of rotation around the axis (from 0° to 360°) and the height variation (from k × 0.5H, where 0 < k < 1);

AngleTorsion=150 : the angular offset between the endpoints of the generating line.

We consider the generating line $\overline{A_1A_2}$, defined by the cylinder's height H, its radius R, and the angular offset θ between its two endpoints.

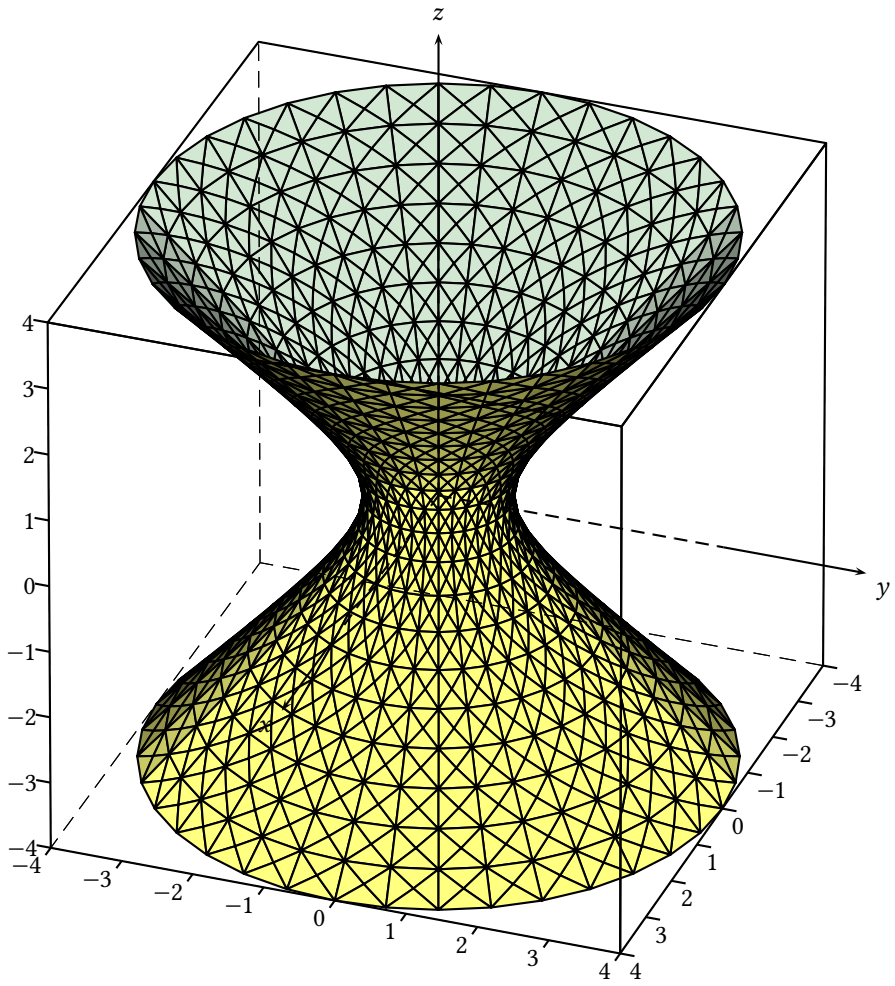


The geometry of a hyperboloid

```
\begin{pspicture}(-8,-6)(8,6)
\psset{solidmemory,viewpoint=50 40 30 rtp2xyz,lightsrc=viewpoint,Decran=50}
\psframe(-5,-6)(5,6)
\pstVerb{/decalage 60 def /H 6 def /Rayon 3 def}%
\psSolid[object=plan,definition=equation,args={[0 0 1 H neg 2 div]},action=none,name=planH]
\psset{plan=planH}
\psProjection[object=cercle,resolution=360,args=0 0 Rayon,linecolor=blue,range=0 360]
\psProjection[object=cercle,resolution=360,args=0 0 Rayon 2 div,linecolor=red,range=0 decalage]
\psSolid[object=plan,definition=equation,args={[0 0 1 H 2 div]},action=none,name=plan0]
\psset{plan=plan0}
\psProjection[object=cercle,resolution=360,args=0 0 Rayon,linecolor=blue,range=0 360]
\psProjection[object=texte,fontsize=20,linecolor=red,PSfont=Symbol,pos=cc,plan=planH,text=q,phi=120](2,1)%
\psPoint(Rayon,0,H 2 div neg){A1}\psdot(A1) \psPoint(Rayon,0,H 2 div){A'1}\psdot(A'1)
\psPoint(0,0,0){O}\psdot(O)
\psLineIIID[linestyle=dashed](Rayon,0,H 2 div)(Rayon neg,0,H 2 div)
\psPoint(0,0,H 2 div neg){O1}\psPoint(0,0,H 2 div){O2}
\psPoint(Rayon decalage cos mul,Rayon decalage sin mul,H 2 div){A2}
\psline[linecolor=red](A1)(A2)\psdots(O1)(O2)(A2)\psline(O2)(A2)
\psline[linestyle=dashed](A1)(A'1) \psline[linestyle=dashed](O1)(O2) \psline(O1)(A1)
\uput[d](A1){$A_1$} \uput[u](A2){$A_2$} \uput[ur](O2){$O_2$} \uput[ur](O1){$O_1$} \uput[ur](O){$O$}
\gridIIID[Zmin=-3,Zmax=3](-3,3)(-3,3)
\end{pspicture}
```

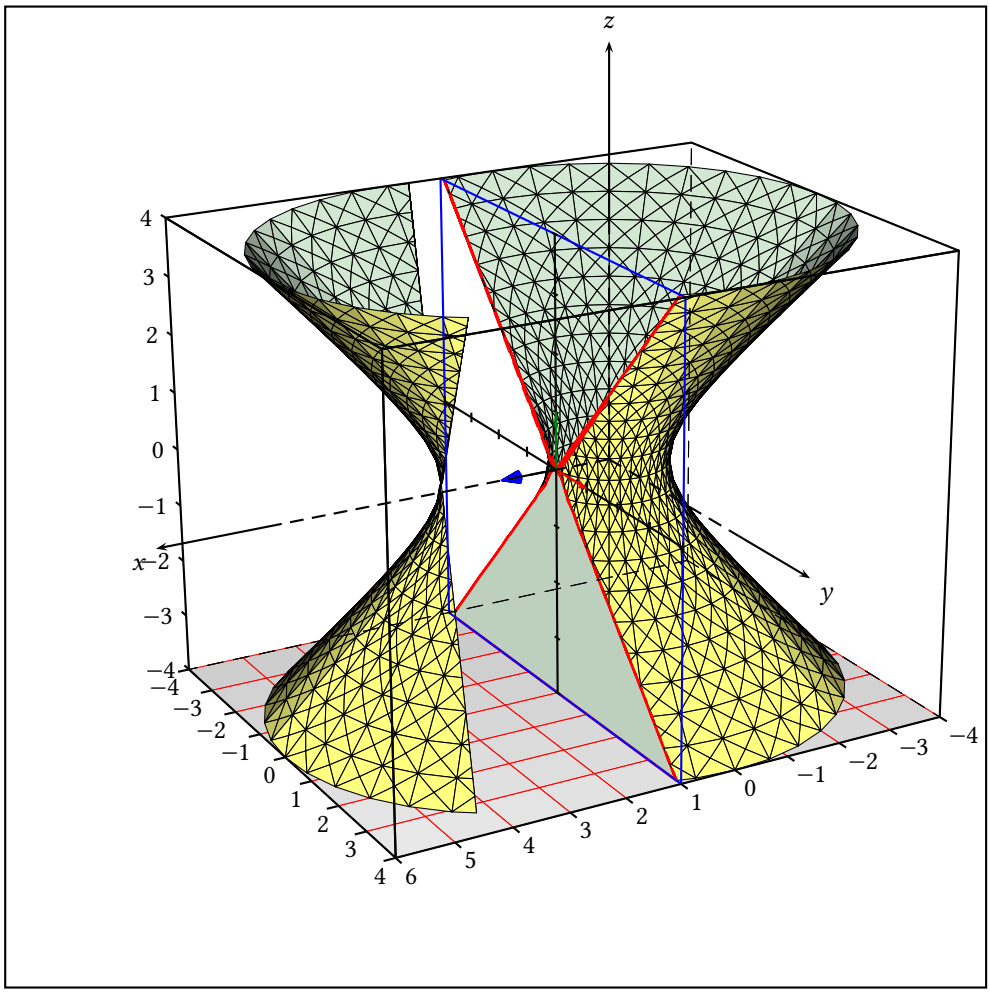
From this, we derive the parametric equations of the hyperboloid:

$$\begin{cases} x = R\big((1-k)\cos(t) + k\cos(t+\theta)\big) \\ y = R\big((1-k)\sin(t) + k\sin(t+\theta)\big) \\ z = (k-\frac{1}{2})H \end{cases}$$



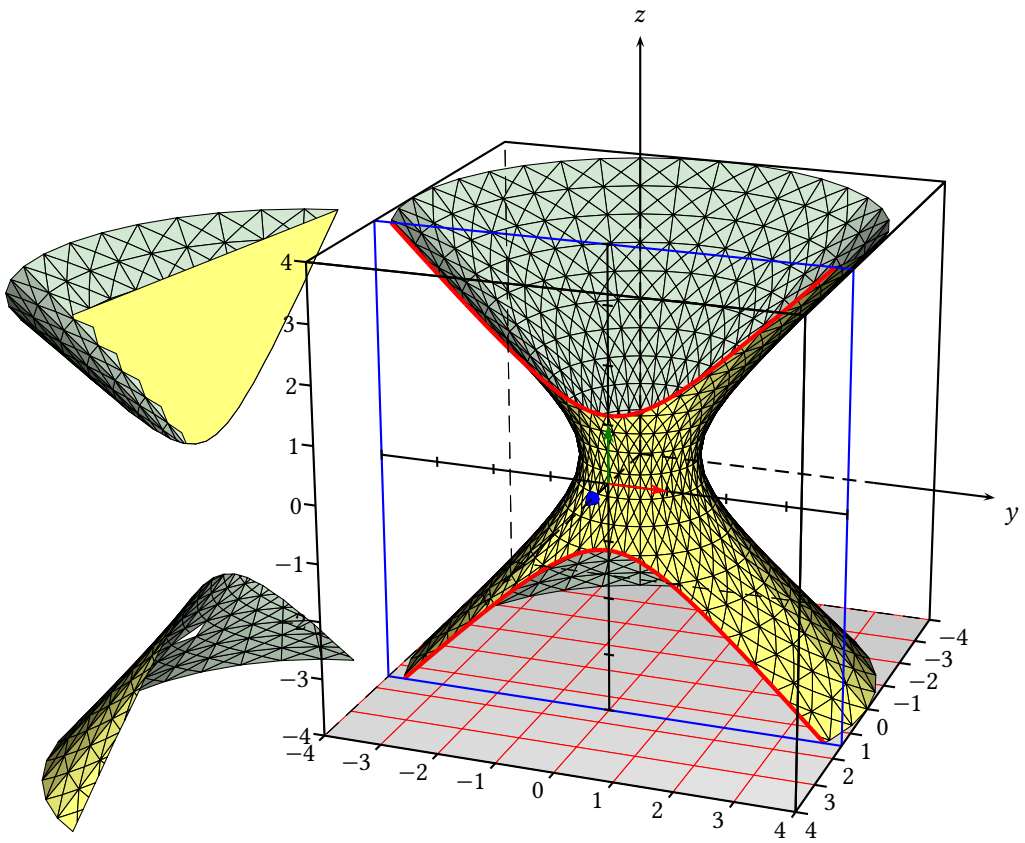
Simple Hyperboloid

```
\begin{pspicture}(-8,-7)(8,7)
\psset{solidmemory,viewpoint=500 20 30 rtp2xyz,lightsrc=viewpoint,Decran=500}
\psHyperboloid[hollow,incolor=yellow!50,fillcolor=ForestGreen!20]
\gridIIID[Zmin=-4,Zmax=4](-4,4)(-4,4)
\end{pspicture}
```



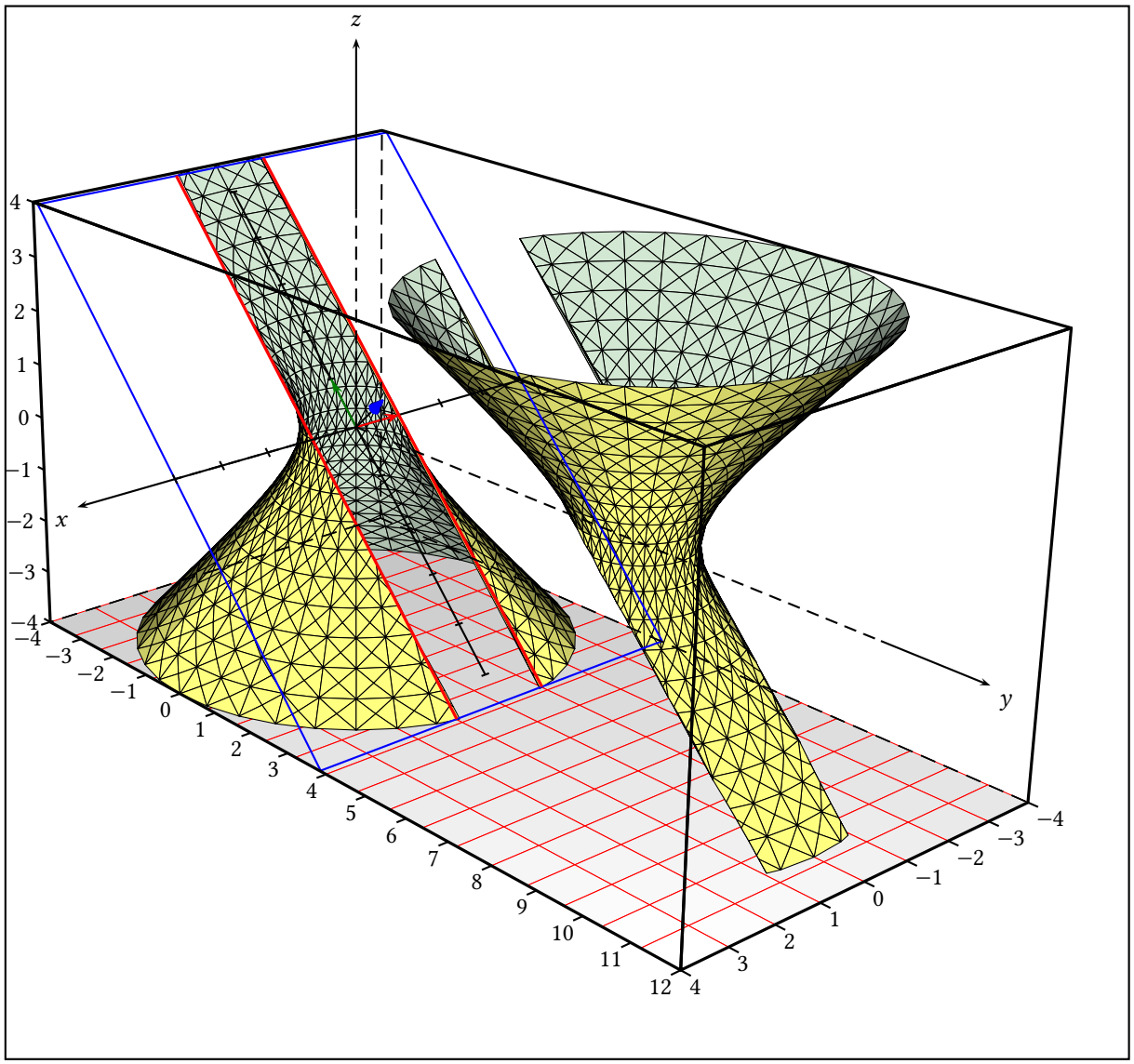
Triangles

```
\begin{pspicture}(-8,-7)(5,6)
\psframe(-8,-7)(5,6)
\psset{solidmemory,viewpoint=50 60 20 rtp2xyz,lightsrc=viewpoint,Decran=40}
\pstVerb{/X0 0.5 def}%
\psSolid[object=grille,base=-4 6 -4 4,ngrid=10 8,linewidth=0.5\pslinewidth,linecolor=red](0,0,-4)%
\psHyperboloid[hollow,R=4,incolor=yellow!50,fillcolor=ForestGreen!20,
  plansepare={[1 0 0 DemiAxeFocal neg]},name=coupeHyperboloidTest,action=none]
\psSolid[object=load,incolor=yellow!50,fillcolor=ForestGreen!20,linewidth=0.5\pslinewidth,
  rm=0,intersectionplan={[1 0 0 DemiAxeFocal neg]},intersectioncolor=(rouge),
  intersectionlinewidth=1,intersectiontype=0,load=coupeHyperboloidTest1]
\psSolid[object=load,incolor=yellow!50,fillcolor=ForestGreen!20,linewidth=0.5\pslinewidth,% rm=0 1 253,
   RotZ=-20,load=coupeHyperboloidTest0](2,0,0)
\psSolid[object=plan,definition=equation,
   args={[1 0 0 DemiAxeFocal neg] 90},linecolor=blue,%   fillcolor=Aquamarine,
   planmarks,base=-4 4 -4 4,showBase,action=draw]
\gridIIID[Zmin=-4,Zmax=4](-4,6)(-4,4)
\end{pspicture}
```



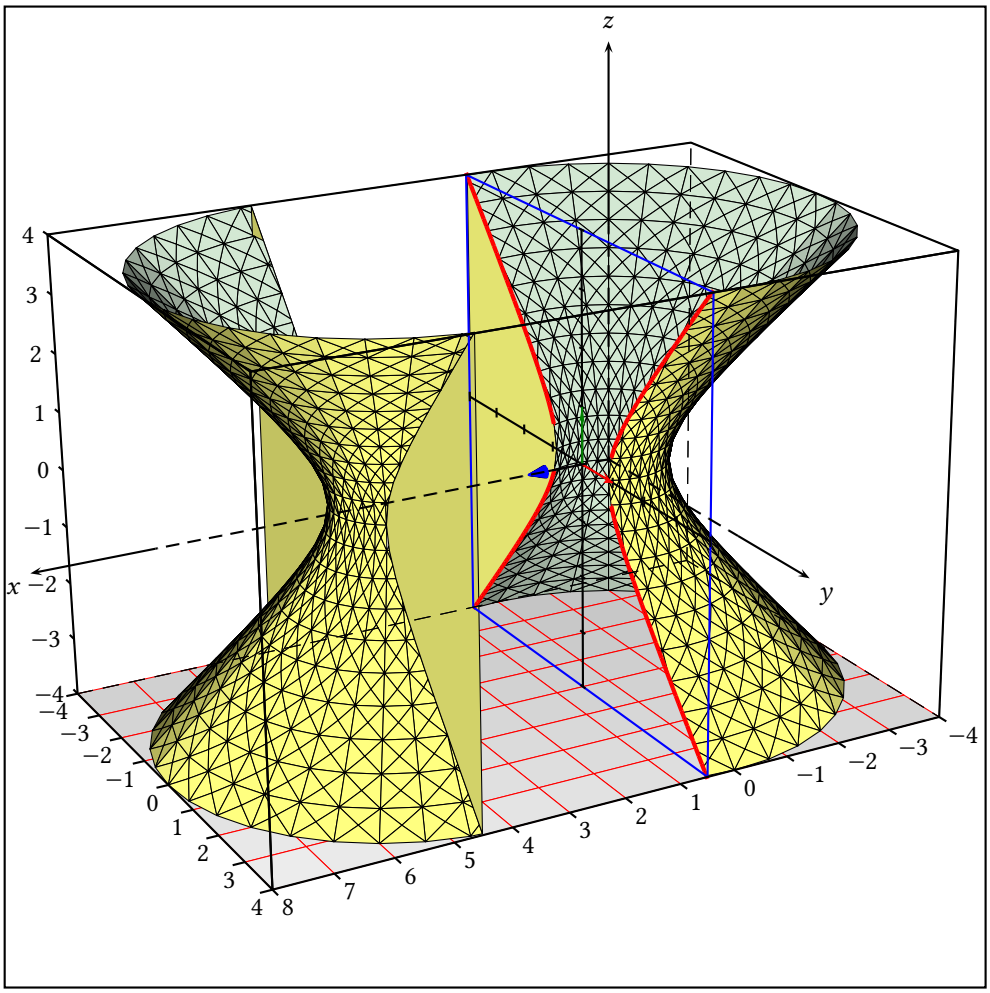
Parabolas

```
\begin{pspicture}(-8,-7)(8,7)
\psset{solidmemory,viewpoint=50 20 20 rtp2xyz,lightsrc=viewpoint,Decran=40}
\pstVerb{/X0 1.5 def}%
\psSolid[object=grille,base=-4 4 -4 4,ngrid=8 8,linewidth=0.5\pslinewidth,linecolor=red](0,0,-4)%
\psHyperboloid[hollow,incolor=yellow!50,fillcolor=ForestGreen!20,plansepare={[1 0 0 X0 neg]},
  name=coupeHyperboloidTest,action=none]
\psSolid[object=load,incolor=yellow!50,fillcolor=ForestGreen!20,linewidth=0.5\pslinewidth,
  rm=0 1,load=coupeHyperboloidTest1]
\defFunction[algebraic]{hyperbol1}(t){X0}{t}{sqrt(((X0^2+t^2)/(DemiAxeFocal^2)-1))*DemiAxeNonFocal}
\defFunction[algebraic]{hyperbol2}(t){X0}{t}{-sqrt(((X0^2+t^2)/(DemiAxeFocal^2)-1))*DemiAxeNonFocal}
\psSolid[object=courbe,linewidth=2\pslinewidth,function=hyperbol1,range=Ymax neg Ymax,r=0,linecolor=red]%
\psSolid[object=courbe,linewidth=2\pslinewidth,function=hyperbol2,range=Ymax neg Ymax,r=0,linecolor=red]%
\psSolid[object=load,incolor=yellow!50,linewidth=0.5\pslinewidth,fillcolor=ForestGreen!20,
  rm=0 1 178 179,load=coupeHyperboloidTest0,RotZ=-100](5,-4,0)
\psSolid[object=plan,definition=equation,args={[1 0 0 -1.5] 90},linecolor=blue,planmarks,
  base=-4 4 -4 4,ngrid={},showBase,action=draw]
\gridIIID[Zmin=-4,Zmax=4](-4,4)(-4,4)
\end{pspicture}
```



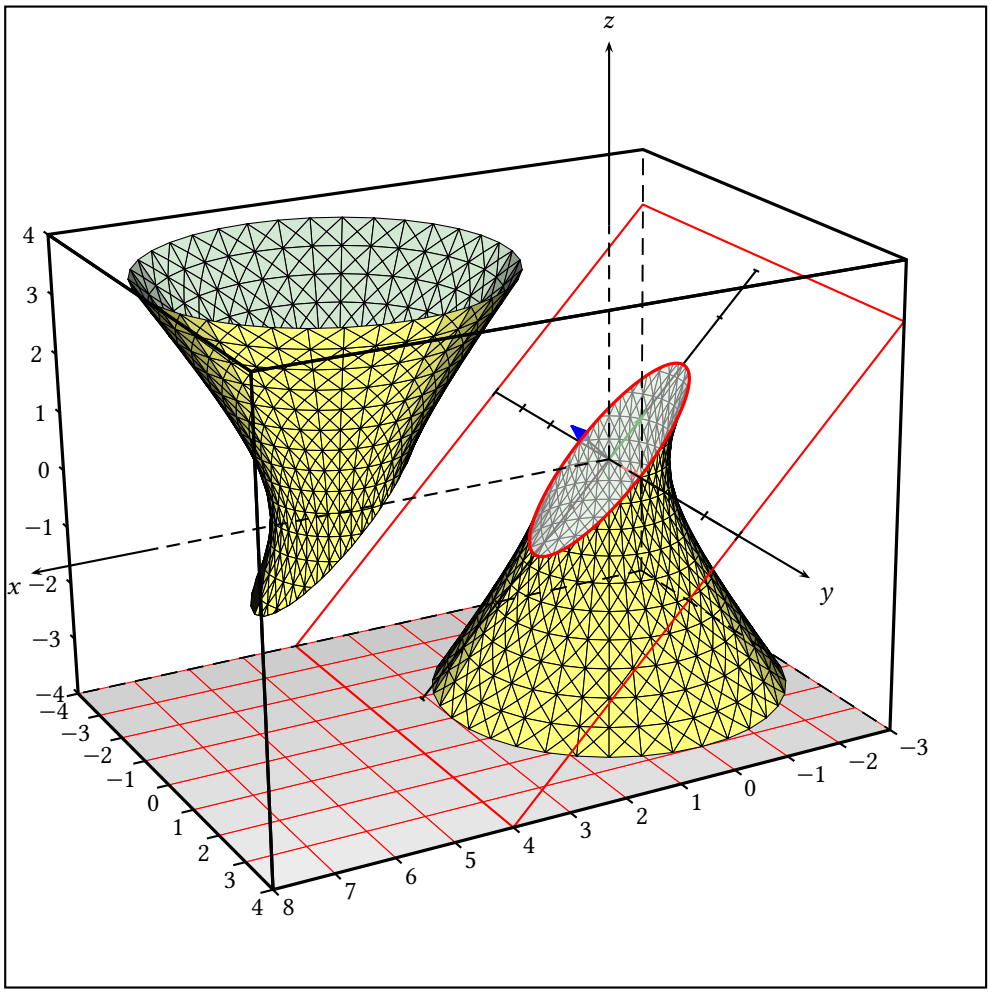
Parallel Lines

```
\begin{pspicture}(-5,-9)(11,6)
\psframe(-5,-9)(11,6)
\psset{solidmemory,viewpoint=50 50 20 rtp2xyz,lightsrc=viewpoint,Decran=40}
\pstVerb{/X0 1.5 def}%
\psSolid[object=grille,base=-4 4 -4 12,ngrid=8 20,linewidth=0.5\pslinewidth,linecolor=red](0,0,-4)%
\psHyperboloid[hollow,incolor=yellow!50,fillcolor=ForestGreen!20,plansepare={[0 1 ConeAsymptote 0]},
  name=coupeHyperboloidTest,action=none]
\psSolid[object=load,incolor=yellow!50,fillcolor=ForestGreen!20,linewidth=0.5\pslinewidth,
  rm=0 1,load=coupeHyperboloidTest1]
\psSolid[object=load,incolor=yellow!50,linewidth=0.5\pslinewidth,fillcolor=ForestGreen!20,% rm=0 1 178 179,
  load=coupeHyperboloidTest0](0,7,0)
\psSolid[object=plan,definition=equation,args={[0 1 ConeAsymptote 0] 180},linecolor=blue,
  planmarks,ngrid=,base=-4 4 -5.55 5.55,showBase,action=draw]
\psSolid[object=plan,definition=equation,args={[0 1 ConeAsymptote 0] 180},base=-5.5 5.5 -5.55 5.55,
   action=none,name=monplan]%
\psProjection[object=droite,plan=monplan,linewidth=1.5\pslinewidth,linecolor=red,args=1 5 1 -5]
\psProjection[object=droite,linewidth=1.5\pslinewidth,plan=monplan,linecolor=red,args=-1 5 -1 -5]
\composeSolid
\gridIIID[Zmin=-4,Zmax=4](-4,4)(-4,12)
\end{pspicture}
```



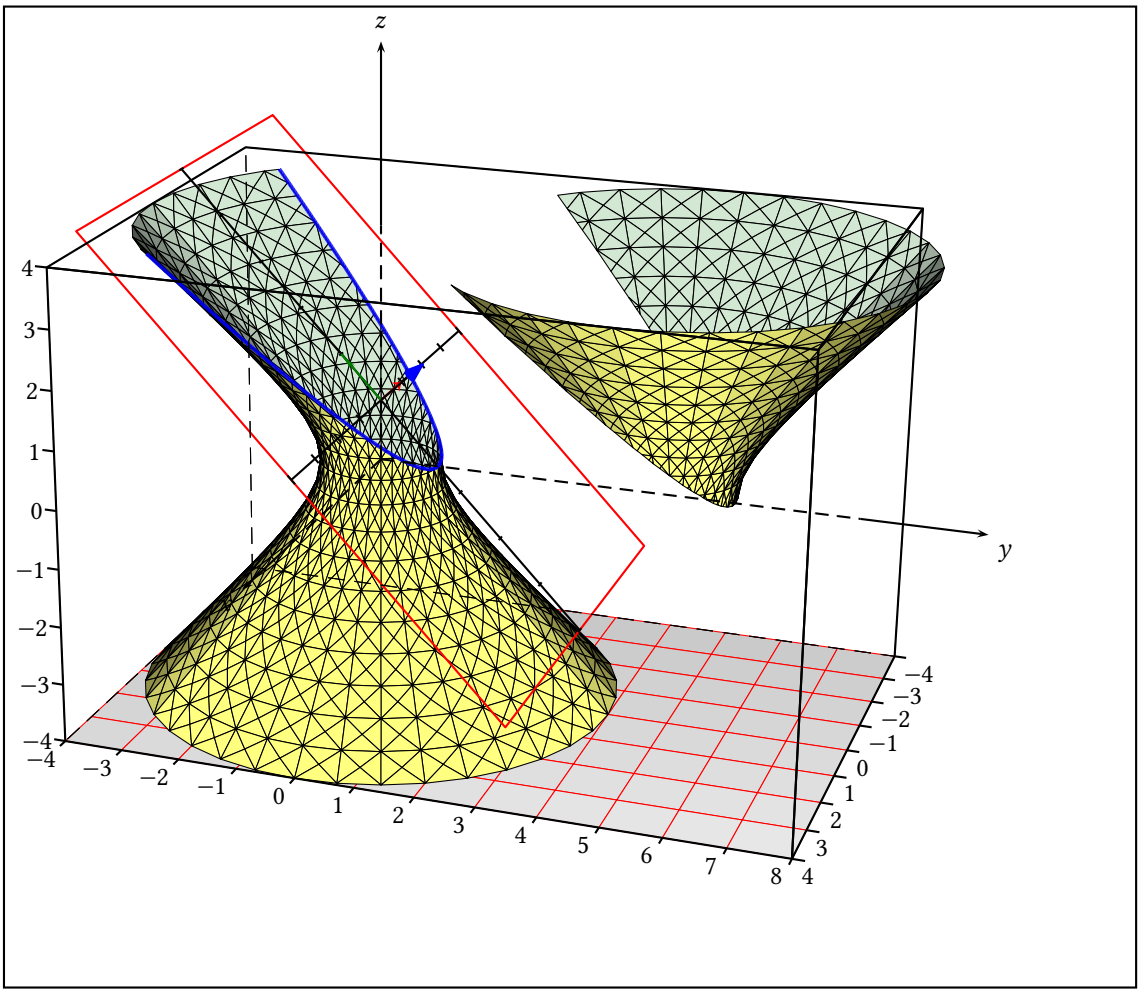
The Hyperbole

```
\begin{pspicture}(-8,-7)(5,6)
\psframe(-8,-7)(5,6)
\psset{solidmemory,viewpoint=50 60 20 rtp2xyz,lightsrc=viewpoint,Decran=40}
\pstVerb{/X0 0.5 def}%
\psSolid[object=grille,base=-4 8 -4 4,ngrid=12 8,linewidth=0.5\pslinewidth,linecolor=red](0,0,-4)%
\psHyperboloid[hollow,incolor=yellow!50,fillcolor=ForestGreen!20,plansepare={[1 0 0 X0 neg]},
   name=coupeHyperboloidTest,action=none]
\psSolid[object=load,incolor=yellow!50,fillcolor=ForestGreen!20,linewidth=0.5\pslinewidth,
   rm=0 1,load=coupeHyperboloidTest1]
\psSolid[object=load,incolor=yellow!50,fillcolor=ForestGreen!20, linewidth=0.5\pslinewidth,
   rm=0 1 333,load=coupeHyperboloidTest0](4,0,0)
\defFunction[algebraic]{hyperbol3}(t){X0}{t}{DemiAxeNonFocal*sqrt((t^2+X0^2)/(DemiAxeFocal^2)-1)}
\defFunction[algebraic]{hyperbol4}(t){X0}{t}{-DemiAxeNonFocal*sqrt((t^2+X0^2)/(DemiAxeFocal^2)-1)}
\psSolid[object=courbe,linewidth=2\pslinewidth,function=hyperbol3,range=Y0 neg -4,r=0,linecolor=red]%
\psSolid[object=courbe,linewidth=2\pslinewidth,function=hyperbol3,range=Y0 4,r=0,linecolor=red]%
\psSolid[object=courbe,linewidth=2\pslinewidth,function=hyperbol4,range=Y0 4,r=0,linecolor=red]%
\psSolid[object=courbe,linewidth=2\pslinewidth,function=hyperbol4,range=Y0 neg -4, r=0,linecolor=red]%
\psSolid[object=plan,definition=equation,args={[1 0 0 -0.5] 90},linecolor=blue,planmarks,
  ngrid=,base=-4 4 -4 4,showBase,action=draw]
\gridIIID[Zmin=-4,Zmax=4](-4,8)(-4,4)
\end{pspicture}
```



The Ellipse

```
\begin{pspicture}(-8,-7)(5,6)
\psframe(-8,-7)(5,6)
\psset{solidmemory,viewpoint=50 60 20 rtp2xyz,lightsrc=viewpoint,Decran=40}
\pstVerb{/X0 0.5 def}%
\psSolid[object=grille,base=-3 8 -4 4,ngrid=11 8,linewidth=0.5\pslinewidth,linecolor=red](0,0,-4)%
\psHyperboloid[hollow,R=3,incolor=yellow!50,fillcolor=ForestGreen!20,
  plansepare={[1 0 1 0]},name=coupeHyperboloidTest,action=none]
\psSolid[object=load,incolor=yellow!50,fillcolor=ForestGreen!20,linewidth=0.5\pslinewidth,rm=0,
  % 1,
  load=coupeHyperboloidTest1]
\psSolid[object=load,incolor=yellow!50,fillcolor=ForestGreen!20,linewidth=0.5\pslinewidth,%rm=0 1,
  load=coupeHyperboloidTest0](5,0,0)
\defFunction[algebraic]{ellipse}(t){DemiAxeFocal*cos(t)}{GrandAxe*sin(t)}{}
\psSolid[object=plan,definition=equation,args={[1 0 1 0] 90},linecolor=red,planmarks,
   base=-4 4 -4 2 sqrt mul 3 2 sqrt mul,showBase,action=draw]%
\defFunction[algebraic]{ellipse}(t){DemiAxeFocal*cos(t)}{GrandAxe*sin(t)}{}
\psSolid[object=plan,definition=equation,args={[1 0 1 0] 90},base=-10 10 -10 10,action=none,
  name=monplan]%
\psProjection[object=courbeR2,plan=monplan,range=0 2 pi mul,resolution=360,linecolor=red,
  fillstyle=solid,opacity=0.5,linewidth=1.5\pslinewidth,function=ellipse]%
\composeSolid
\gridIIID[Zmin=-4,Zmax=4](-3,8)(-4,4)
\end{pspicture}
```



The Parabola

```
\begin{pspicture}(-5,-7)(10,6)
\psframe(-5,-7)(10,6)
\psset{solidmemory}
\psset{viewpoint=50 20 20 rtp2xyz,lightsrc=viewpoint,Decran=40}
\pstVerb{/X0 0.5 def}%
\psSolid[object=grille,base=-4 4 -4 8,ngrid=8 12,linewidth=0.5\pslinewidth,linecolor=red](0,0,-4)%
\psHyperboloid[hollow,R=4,incolor=yellow!50,fillcolor=ForestGreen!20,plansepare={[0 1 ConeAsymptote -1]},
   name=coupeHyperboloidTest,action=none]
\psSolid[object=load,incolor=yellow!50,fillcolor=ForestGreen!20,linewidth=0.5\pslinewidth,
   rm=0,load=coupeHyperboloidTest1]
\psSolid[object=load,incolor=yellow!50,fillcolor=ForestGreen!20,linewidth=0.5\pslinewidth,
   rm=236,load=coupeHyperboloidTest0](0,5,0)
\psSolid[object=plan,definition=equation,args={[0 1 ConeAsymptote -1] 180},linecolor=red,
   planmarks,base=-4 4 -5 5,showBase,action=draw]%
\defFunc-
↪  tion[algebraic]{parabole1}(t){DemiAxeFocal*sqrt(1+(t/DemiAxeNonFocal)^2-((1-ConeAsymptote*t)/DemiAxeFocal)^2)}{1-ConeAsymptote
\defFunc-
↪  tion[algebraic]{parabole2}(t){-DemiAxeFocal*sqrt(1+(t/DemiAxeNonFocal)^2-((1-ConeAsymptote*t)/DemiAxeFocal)^2)}{1-ConeAsymptot
\psSolid[object=courbe,r=0,range=0 4,linecolor=blue,linewidth=0.05,resolution=360,function=parabole1]%
\psSolid[object=courbe,r=0,range=0 4,linecolor=blue,linewidth=0.05,resolution=360,function=parabole2]%
\composeSolid
\gridIIID[Zmin=-4,Zmax=4](-4,4)(-4,8)
\end{pspicture}
```

References

[1] WIKIPEDIA. *Uniform tilings in hyperbolic plane*. URL: https://en.wikipedia.org/wiki/Uniform_tilings_in_hyperbolic_plane (visited on 05/30/2026).

[2] David Eppstein. *The Geometry Junkyard. Tilings of Hyperbolic Spaces*. URL: https://ics.uci.edu/~eppstein/junkyard/hypertile.html (visited on 05/30/2026).

[3] Patrick Fradin. *Pavages hyperboliques*. URL: https://melusine.eu.org/syracuse/texgraph/exemples/hpavages/ (visited on 05/28/2026).

[4] Frank Mittelbach et al. *The LaTeX Graphics Companion*. 2nd ed. Reprint. Heidelberg and Berlin: Lehmanns Media, 2022.

Index