ASYMPTOTIC ANALYSIS OF ELASTIC SHELLS

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Shells and their assemblages are found in a wide variety of elastic structures: Cooling towers, aircrafts, car bodies, sails, hulls of vessels, etc. These shells are, and therefore should be *in principle* studied as, *three–dimensional* bodies. The *"small"* thickness of a shell makes it however natural to "replace" the genuine three–dimensional model by a simpler *two–dimensional* model, i.e., one that is posed over the middle surface of the shell.

Not only is this replacement intuitively "natural", but it becomes a *necessity* when *numerical methods* must be devised for computing approximate displacements and stresses: Any reasonably accurate *three–dimensional* discretization necessarily involves an outstandingly large number of unknowns, which renders it prohibitively expensive, and at any rate its implementation requires extreme care. By contrast, the situation is on much safer grounds as regards the application of approximate methods to *two–dimensional shell models*, although there remain challenging problems, such as that of efficiently coping with the onset of *locking*.

Since two–dimensional shell models are for these reasons by and large preferred, two major, and closely related, questions naturally arise:

How to derive two–dimensional shell models in a rational manner from three-dimensional elasticity? This first question is one in *asymptotic analysis*; it consists in analyzing the behavior of the three–dimensional solution as the thickness (the "small" parameter) approaches zero.

How to choose among the various available two–dimensional shell models in a given physical situation, so that the chosen one be indeed a "good" approximation of the three-dimensional model it "replaces"? This second question is of paramount *practical* importance, for it makes no sense to devise methods for accurately approximating the solution of a "wrong" model!

Our purpose is to describe recent progress on these questions. Our discussion will focus in particular on three well known two–dimensional shell models: the *bending* and *membrane* models, and *W.T. Koiter's model.*