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ON THE SYMPLECTIC STRUCTURE OF SOME NATURAL BUNDLES

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ABSTRACT. We study geometrical constructions of a symplectic structure on the bundles of k-dimensional velocities of order r by means of a symplectic structure on the base manifold. Then we determine all natural symplectic structures on the bundles $T^r_k T^* M$.

1. Introduction

An almost symplectic structure on a manifold M is a 2–form ω on M such that the mapping $\omega_L : TM \to T^*M$ defined by $\omega_L(y) = i_y\omega$ is a vector bundle isomorphism. An almost symplectic structure ω on M is called a *symplectic structure* if $\mathrm{d}\omega = 0$.

The aim of this paper is twofold:

First, we discuss geometrical constructions of a symplectic structure on some product preserving bundles FM by means of a symplectic structure on the original manifold M. For example, a symplectic structure ω on M induces two canonical symplectic structures ω^C and ω^A on the tangent bundle TM of M. The tangent functor T is the simplest example of a product preserving bundle functor. A more general example is the functor T^r_k of k–dimensional velocities of order r defined by

$$T^r_k M = J^r_0(\mathbb{R}^k, M), \quad T^r_k f(j^r_0 g) = j^r_0(f \circ g) \tag{1}$$

for an arbitrary smooth manifold M and arbitrary smooth mapping $f : M \to N$. In particular, $TM = T^1_1 M$ is the tangent bundle and if $\dim M = m$, then $P^r M = \mathrm{inv} T^r_m M$ is the r-h order frame bundle. We remark that the functor T^r_k has been used in the geometrical approach to the analytical mechanics, see e.g. [3].

Second, we discuss the existence of a natural symplectic structure on the bundle $T^r_k T^* M$. We also classify all such natural symplectic structures.

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2. Preliminaries

I. Product preserving bundle functors. Let $\mathcal{M}f$ be the category of all smooth manifolds and all smooth maps and $\mathcal{FM}$ be the category of fibered manifolds and fibered manifold morphisms. We say that the bundle functor $F : \mathcal{M}f \to \mathcal{FM}$ preserves products, if the diagram

$$F(M_1) \xleftarrow{F(\mathrm{pr}_1)} F(M_1 \times M_2) \xrightarrow{F(\mathrm{pr}_2)} F(M_2)$$

is always a product diagram. For example, the functor T_k^r of k-dimensional velocities of order r is product preserving bundle functor. The best known example of a product preserving bundle functor is the Weil functor T^A of A-velocities. For a definition of T^A see e.g [11]. By [11], Weil functors represent a general model for all product preserving bundle functors. In fact, Kainz and Michor and independently Eck and Luciano have proved that for every product preserving bundle functor F there is a Weil algebra A such that $F = T^A$. In what follows we shall use the following general assertion from [11].

Proposition 1. *For all product preserving bundle functors F and G there is a canonical natural equivalence $\kappa : F \circ G \to G \circ F$.*

In particular, if $F = G = T$, then $\kappa_M : TTM \to TTM$ is the canonical involution of the iterated tangent bundle.

II. The natural equivalence $T_1^r T^* \to T^* T_1^r$. By Cantrijn, Crampin, Sarlet and Saunders [1], there is a natural equivalence

$$s : T_1^r T^* \to T^* T_1^r \, . \tag{2}$$

If M is a smooth manifold, then the corresponding isomorphism $T_1^r T^* M \to T^* T_1^r M$ will be denoted by s_M. For $r = 1$, s_M coincides with the well known natural equivalence $TT^*M \to T^*TM$ by Tulczyjev and Modugno and Stefani. In [6] we have proved that for $k > 1$ there is no natural equivalence $T_k^1 T^* \to T^* T_k^1$. Further, in [7] we have proved the following more general assertion.

Proposition 2. *For $k > 1$ there is no natural equivalence $T_k^r T^* \to T^* T_k^r$.*

III. The complete lift. Let $T^{(r,s)}$ be the functor corresponding to the bundle $T^{(r,s)}M \to M$ of all tensors of the type (r,s) on M. If F is a natural bundle, then the natural operator $T^{(r,s)} \to T^{(r,s)}F$ transforming tensor fields of the type (r,s) on M into tensor fields of the same type on FM will be called a lifting. In what follows we shall need a specific type of such natural operator, which will be called the complete lift.

If $f : M \to \mathbb{R}$ is a smooth function, then the complete lift of f to $T_1^r M$ is a function $f^C : T_1^r M \to \mathbb{R}$ defined by

$$f^C(j_0^r \gamma) = \frac{\partial^r}{\partial t^r}\bigg|_0 (f \circ \gamma) \, . \tag{3}$$

Denoting by $\widetilde{f} = T_1^r f : T_1^r M \to T_1^r \mathbb{R}$, then the complete lift $f^C : T_1^r M \to \mathbb{R}$ is nothing else but the $(r+1)$-th component of $\widetilde{f}$.

If X is a vector field on M, then the complete lift of X to $T_1^r M$ is defined as the flow prolongation of X, $X^C = \mathcal{T}_1^r X$. Clearly, $X^C = \kappa_M \circ T_1^r X$, where $\kappa_M : T_1^r TM \to TT_1^r M$ is the exchange mapping from Proposition 1.

Finally, let ω be a tensor field of the type $(0,2)$ on M. Then ω can be interpreted as a bilinear mapping $\omega : TM \times_M TM \to \mathbb{R}$. Applying T_1^r we obtain $T_1^r \omega : T_1^r TM \times_{T_1^r M} T_1^r TM \to T_1^r \mathbb{R}$ and the composition with $\kappa_M : T_1^r TM \to TT_1^r M$ yields a map

$$\widetilde{\omega} = T_1^r \omega \circ (\kappa_M^{-1} \times \kappa_M^{-1}) : TT_1^r M \times_{T_1^r M} TT_1^r M \to T_1^r \mathbb{R} \,. \tag{4}$$

Let $\widetilde{\omega}_{r+1} : TT_1^r M \times_{T_1^r M} TT_1^r M \to \mathbb{R}$ be the $(r+1)$-th component of $\widetilde{\omega}$. Clearly, $\widetilde{\omega}_{r+1}$ is a bilinear mapping, so that it determines a $(0,2)$-tensor field ω^C on $T_1^r M$, which will be called a complete lift of ω to $T_1^r M$.

If $X : M \to TM$ is a vector field and $\alpha : M \to T^*M$ is a 1-form on M, then $\langle \alpha, X \rangle : M \to \mathbb{R}$ denotes the evaluation mapping. By a direct evaluation we prove the following two lemmas.

Lemma 1. $\omega^C(X^C, Y^C) = (\omega(X,Y))^C$

Lemma 2. *Let $s_M : T_1^r T^*M \to T^*T_1^r M$ be the natural equivalence (2), let α be an arbitrary 1-form on M and let X be a vector field on M. Then*

$$\left\langle s_M T_1^r \alpha, X^C \right\rangle = \langle \alpha, X \rangle^C \,. \tag{5}$$

The following properties of the complete lift ω^C were proved in [8].

Proposition 3. *Let ω be a tensor field of type $(0,2)$ on M.*

(a) *If ω is symmetric (respectively skew-symmetric), then ω^C is also symmetric (respectively skew symmetric).*
(b) *If ω is a closed 2-form on M, then ω^C is a closed 2-form on $T_1^r M$.*
(c) *If ω is an almost symplectic structure on M, then ω^C is an almost symplectic structure on $T_1^r M$. If ω is a symplectic structure, then so is ω^C.*

3. Liftings of a symplectic structure to the bundle of k-dimensional velocities of order r

The aim of this section is to study geometrical constructions of a symplectic structure on $T_k^r M$ by means of a symplectic structure on M.

I. The simplest particular case is the tangent bundle TM. If ω is a 2-form on M, then $\omega_L : TM \to T^*M$ will denote the linear mapping induced by ω, defined by $\omega_L(y) = i_y \omega$. We have

Proposition 4. *Let ω be a symplectic form on M and let $\omega_L : TM \to T^*M$ be the corresponding isomorphism. Let $\kappa_M : TTM \to TTM$ be the canonical involution and $s_M : TT^*M \to T^*TM$ be the natural equivalence (2). Then the complete lift ω^C is a symplectic form on TM and $(\omega^C)_L : TTM \to T^*TM$ is of the form $(\omega^C)_L = s_M \circ T\omega_L \circ \kappa_M$.*

Proof. It follows directly from Proposition 3 and from Proposition 1 in [5]. □

Let ω be a symplectic form on M, Ω be the canonical symplectic form on T^*M and $\omega_L : TM \to T^*M$ be the isomorphism induced by ω. Then the pullback

$$\omega^A = (\omega_L)^*\Omega \tag{6}$$

is another symplectic form on the tangent bundle. The 2-form ω^A is called the antisymmetric lift of ω to TM. In coordinates, if $\omega = \omega_{ij}dx^i \otimes dx^j$ is a $(0,2)$-tensor field on M, then

$$\omega^C = \left(\frac{\partial \omega_{ij}}{\partial x^k} y^k\right) dx^i \otimes dx^j + \omega_{ij} dx^i \otimes dy^j + \omega_{ij} dy^i \otimes dx^j,$$

$$\omega^A = \left(\frac{\partial \omega_{jm}}{\partial x^i} y^m\right) dx^i \wedge dx^j - \omega_{ij} dx^i \wedge dy^j$$

where $(x^i, y^i = dx^i)$ are the induced coordinates on TM.

Remark 1. In [5] we have determined all first order natural operators $T^{(0,2)} \to T^{(0,2)}T$ transforming general $(0,2)$-tensor fields on M into $(0,2)$-tensor fields on TM. All natural operators of such a type were determined in [12] by Mikulski. If we restrict ourselves to 2-forms only, we have: *The complete lift ω^C and the antisymmetric lift ω^A form a basis of the module of all natural operators transforming 2-forms on M into symplectic forms on TM.*

Remark 2. In [9], J. Janyška has determined all first order natural operators transforming Riemannian metrics on M into symplectic forms on TM.

II. We first show a generalization of Proposition 4.

Proposition 5. *Let ω be a symplectic form on M and $\omega_L : TM \to T^*M$ be the corresponding isomorphism. Let $\kappa_M : TT_1^rM \to T_1^rTM$ and $s_M : T_1^rT^*M \to T^*T_1^rM$. Then the complete lift ω^C is a symplectic form on T_1^rM and $(\omega^C)_L : TT_1^rM \to T^*T_1^rM$ is of the form $(\omega^C)_L = s_M \circ T_1^r\omega_L \circ \kappa_M$.*

Proof. By Proposition 3, the complete lift ω^C is a symplectic form on T_1^rM. The rest follows directly from Lemma 1 and Lemma 2. □

Lemma 3.

I. *Let M and T^AM be symplectic manifolds. Then there is a canonical isomorphism $s_M : T^AT^*M \to T^*T^AM$.*

II. *Conversely, if M is a symplectic manifold, then every isomorphism $s_M : T^AT^*M \to T^*T^AM$ induces a symplectic structure on T^ATM.*

Proof. I. Denote by $\omega_L : TM \to T^*M$ and $G_L : TT^AM \to T^*T^AM$ isomorphisms generated by the symplectic forms ω on M and G on T^AM, respectively. Then $T^A(\omega_L) : T^ATM \to T^AT^*M$ is an isomorphism as well. Denoting by $\kappa_M : T^ATM \to TT^AM$ the exchange isomorphism from Proposition 1, we may put $s_M = G_L \circ \kappa_M \circ (T^A(\omega_L))^{-1}$.
II. Let $\omega_L : TM \to T^*M$ be the isomorphism induced by a symplectic form ω on M. Then $T^A(\omega_L) : T^ATM \to T^AT^*M$ is an isomorphism as well, so that the composition $f := s_M \circ T^A(\omega_L) : T^ATM \to T^*T^AM$ is an isomorphism. If Ω is the canonical symplectic form on T^*T^AM, then the pull-back $f^*(\Omega)$ is a symplectic form on T^ATM. $\square$

4. Natural symplectic structures on $T^r_kT^*M$

Definition. Let F be a natural bundle. A natural symplectic form ω on F is defined as a system of symplectic forms $\omega_M : FM \to \Lambda^2T^*(FM)$ for every m-dimensional manifold M satisfying $\Lambda^2T^*Ff \circ \omega_M = \omega_N \circ Ff$ for all local diffeomorphisms $f : M \to N$.

By Cantrijn, Crampin, Sarlet and Saunders, the manifold $T^r_1T^*M$ is endowed with a natural symplectic structure. This symplectic structure is the pull-back of the canonical symplectic structure of the cotangent bundle $T^*(T^r_1M)$ with respect to the isomorphism $s_M : T^r_1T^*M \to T^*T^r_1M$. We have

Proposition 6. *There is a natural symplectic structure on $T^r_kT^*M$ if and only if $k = 1$.*

Proof. Clearly, the canonical isomorphism $s_M : T^r_1T^*M \to T^*T^r_1M$ induces the natural symplectic structure on $T^r_1T^*M$. In what follows we prove, that for $k > 1$ there is no isomorphism $TT^r_kT^*M \to T^*T^r_kT^*M$ over the identity of $T^r_kT^*M$.

The canonical coordinates (x^i) on M induce the additional coordinates (p_i) on T^*M and (x^i_γ) on T^r_kM. Then the canonical coordinates on $TT^r_kT^*M$ are $(x^i, p_i, x^i_\gamma, p_{i,\gamma}, X^i = dx^i, P_i = dp_i, X^i_\gamma = dx^i_\gamma, P_{i,\gamma} = dp_{i,\gamma})$. Finally, expressions $r_idx^i + s^\gamma_i dx^i_\gamma + \alpha^i dp_i + \beta^{i,\gamma}dp_{i,\gamma}$ define the additional coordinates $(r_i, s^\gamma_i, \alpha^i, \beta^{i,\gamma})$ on $T^*T^r_kT^*M$. Consider first the case $r = 1$. Denote by G^s_m the group of all invertible s-jets of $\mathbb{R}^m$ into $\mathbb{R}^m$ with source and target zero. By the general theory of natural operations in differential geometry, [11], natural transformations $TT^1_kT^* \to T^*T^1_kT^*$ are in the canonical bijection with G^3_m-equivariant maps of the corresponding standard fibres. Denoting by $(a^i_j, a^i_{jk}, a^i_{jk\ell})$ the canonical coordinates on G^3_m, coordinates of the inverse element will be denoted by a tilde. One evaluates easily the following transformation laws

$$\overline{p}_i = \tilde{a}^j_i p_j, \quad \overline{x}^i_\gamma = a^i_j x^j_\gamma, \quad \overline{\beta}^{i,\gamma} = a^i_j \beta^{j,\gamma}, \quad \overline{p}_{i,\gamma} = \tilde{a}^j_i p_{j,\gamma} + \tilde{a}^j_{ik} a^k_\ell x^\ell_\gamma p_j,$$
$$\overline{X}^i = a^i_j X^j, \; \overline{X}^i_\gamma = a^i_j X^j_\gamma + a^i_{jk} x^j_\gamma X^k, \qquad \overline{P}_i = \tilde{a}^j_i P_j + \tilde{a}^j_{ik} a^k_\ell p_j X^\ell .$$

Further, on the kernel of the jet projection $G^3_m \to G^2_m$ we have $\overline{P}_{i,\gamma} = P_{i,\gamma} + \tilde{a}^j_{ik\ell} x^k_\gamma p_j X^\ell$. Consider first

$$\beta^{i,\gamma} = \beta^{i,\gamma}(p_i, x^i_\gamma, p_{i,\gamma}, X^i, P_i, X^i_\gamma, P_{i,\gamma}) .$$

Using equivariance on the kernel of the jet projection $G^3_m \to G^1_m$ we obtain that $\beta^{i,\gamma}$ are independent of $P_{i,\gamma}$. As we look for linear natural transformations only, $\beta^{i,\gamma}$ are of the form

$$\beta^{i,\gamma} = A^\gamma X^i + B^{\gamma\delta} X^i_\delta + C^{\gamma\delta\rho} x^i_\delta x^j_\rho P_j \,.$$

Then the full equivariance yields that $B = 0$, $C = 0$, so that

$$\beta^{i,\gamma} = A^\gamma X^i \tag{7}$$

for all $\gamma = 1, \dots, k$. Clearly, if $k > 1$, then (7) are not equations of an isomorphism. One verifies directly that in the general case of an arbitrary $r \geq 1$ we have $\overline{\beta}^{i,\gamma} = a^i_j \beta^{j,\gamma}$ for all γ. On the other hand, among $(X^i, P_i, X^i_\gamma, P_{i,\gamma})$, only X^i have the tensorial transformation law $\overline{X}^i = a^i_j X^j$. Quite similarly to the case $r = 1$ we show that $\beta^{i,\gamma} = A^\gamma X^i$ for all $\gamma = (\gamma_1, \dots, \gamma_k)$, $|\gamma| = (\gamma_1 + \dots + \gamma_k) \leq r$. If $k > 1$, then this is not the component of an isomorphism $TT^r_k T^*M \to T^*T^r_k T^*M$. $\square$

Using coordinates we verify easily

Proposition 7. *Let $\omega = dp_i \wedge dx^i$ be the canonical symplectic form of the cotangent bundle T^*M and let $s_M : T^r_1 T^*M \to T^*T^r_1 M$ be the natural equivalence (2). Denote by ω^C the complete lift of ω to $T^r_1(T^*M)$ and by ω^A the antisymmetric lift of ω to $T(TM)$. Let Ω be the canonical symplectic form of the cotangent bundle $T^*(T^r_1 M)$. Then*

$$s^*_M \Omega = \omega^C \tag{8}$$

and for $k = 1$ we have $s^*_M \Omega = \omega^C = \omega^A$.

At the end we prove that the symplectic form $s^*_M \Omega$ from Proposition 7 is in a certain sense the only natural one.

Definition. Let F be a natural bundle. A natural function g on F is a system of functions $g_M : FM \to \mathbb{R}$ for every m-manifold M satisfying $g_M = g_N \circ Ff$ for all local diffeomorphisms $f : M \to N$.

Definition. A natural vector field ξ on F is a system of vector fields $\xi_M : FM \to TFM$ for every m-manifold M satisfying $TFf \circ \xi_M = \xi_N \circ Ff$ for all local diffeomorphisms $f : M \to N$.

Denote by $q_M : T^*FM \to FM$ the cotangent bundle projection. Then for every natural vector field ξ on F, the maps

$$\widetilde{\xi}_M : T^*FM \to \mathbb{R}, \quad \widetilde{\xi}_M(w) = \langle \xi_M(q_M w), w \rangle$$

form a natural function $\widetilde{\xi}$ on T^*F. I. Kolář, [10], has determined all natural functions on $T^*T^r_1$. Let L_M be the generalized Liouville vector field on $T^r_1 M$ induced by the reparametrization $x(t) \mapsto x(kt)$, $0 \neq k \in \mathbb{R}$ of a curve $x : \mathbb{R} \to M$

and let $Q_M : TT_1^r M \to TT_1^r M$ be the natural linear morphism introduced by de León and Rodrigues in [2]. Such tensor fields of type $(1,1)$ are called affinors. We remark that Q_M is the particular case of the natural affinor $TT^A M \to TT^A M$, which was defined in [11] for arbitrary Weil bundle $T^A M$. Denote by (x^i) the canonical coordinates on $\mathbb{R}^m$ and by $(y_1^i, \dots, y_r^i)$ the induced coordinates on $T_1^r \mathbb{R}^m$. Then the coordinate form of $Q_{\mathbb{R}^m}$ is

$$(dx^i, dy_1^i, \dots, dy_r^i) \mapsto (0, dx^i, \dots, dy_{r-1}^i)\,.$$

By I. Kolář, [11], all natural vector fields on T_1^r form an r-parameter family linearly generated by $L_1 = L$, $L_2 = Q \circ L, \dots, L_r = Q^{r-1} \circ L$.

Proposition 8 ([10]). *All natural functions on $T^*T_1^r$ are of the form $h(\widetilde{L}_1, \dots, \widetilde{L}_r)$ for all smooth functions $h \in C^\infty(\mathbb{R}^r, \mathbb{R})$.*

Denoting by $q_i dx^i + q_i^1 dy_1^i + \dots + q_i^r dy_r^i$ the additional coordinates on $T^*T_1^r\mathbb{R}^m$, the coordinate form of the natural functions $\widetilde{L}_1, \dots, \widetilde{L}_r$ on $T^*T_1^r\mathbb{R}^m$ is

$$\begin{aligned} & q_i^1 y_1^i + \dots + r q_i^r y_r^i\,, \\ & q_i^2 y_2^i + \dots + (r-1) q_i^r y_r^i\,, \\ & \quad \vdots \\ & q_i^r y_1^i\,. \end{aligned} \tag{9}$$

Taking into account the canonical isomorphism $s_M : T_1^r T^* M \to T^* T_1^r M$, the natural functions $\widetilde{L}_1, \dots, \widetilde{L}_r$ on $T^*T_1^r$ induce natural functions $\widetilde{B}_1, \dots, \widetilde{B}_r$ on $T_1^r T^*$, $(\widetilde{B}_i)_M = s_M \circ (\widetilde{L}_i)_M$. We have

Proposition 9. *All natural symplectic forms on $T_1^r T^*$ are of the form*

$$f(\widetilde{B}_1, \dots, \widetilde{B}_r) \cdot s_M^* \Omega \tag{10}$$

where $f : \mathbb{R}^r \to \mathbb{R}$ is an arbitrary smooth function and $s_M^* \Omega$ is the symplectic form from Proposition 7.

Proof. We identify a symplectic form on $T_1^r T^* M$ with an isomorphism $TT_1^r T^* M \to T^* T_1^r T^* M$ over the identity of $T_1^r T^* M$. Consider first the case $r = 1$. In [4] we have determined all natural transformations $TTT^*M \to TT^*TM$. Using the natural equivalence $s : TT^* \to T^*T$ and the classical natural equivalence between TT^* and T^*T^*, we can determine all natural transformations $TTT^*M \to T^*TT^*M$ over the identity of TT^*M. Denote by $(x^i, p_i, z^i = dx^i, q_i = dp_i)$ the canonical coordinates on TT^*M, by (X^i, P_i, Z^i, Q_i) the additional coordinates on TTT^*M and by $r_i dx^i + s_i dz^i + \alpha^i dp_i + \beta^i dq_i$ the additional coordinates on T^*TT^*M. By [4], all natural transformations $TTT^*M \to T^*TT^*M$ over the identity of TT^*M are of the form

$$\begin{aligned} \beta^i &= B X^i\,, \\ \alpha^i &= B Z^i + H z^i + K X^i\,, \\ s_i &= -B P_i + F p_i\,, \\ r_i &= -B Q_i - H q_i - K P_i + F q_i + L p_i\,, \end{aligned} \tag{11}$$

where B, H, K, F and L are arbitrary smooth functions of the invariants $I_1 = p_i z^i$, $I_2 = p_i X^i$, $I_3 = p_i Z^i + P_i z^i$, $I_4 = p_i Z^i + q_i X^i$. If we restrict ourselves to linear natural transformations only, we find that all such natural transformations over the identity of TT^*M form a free module over $C^\infty(I_1)$. Clearly, $I_1 = p_i z^i$ is in fact the coordinate form of $\widetilde{B}_1$. Further, the only natural equivalence among above linear natural transformations corresponds exactly to the symplectic form $s_M^* \Omega$ on TT^*M, so that our assertion is proved for $r = 1$. The proof in the general case is quite similar: it suffices to determine all isomorphisms $TT_1^r T^*M \to T^*T_1^r T^*M$ over the identity of $T_1^r T^*M$. By Proposition 8, all natural functions on $T_1^r T^*$ are of the form $h(\widetilde{B}_1, \dots, \widetilde{B}_r)$ for all $h \in C^\infty(\mathbb{R}^k, \mathbb{R})$. Using the canonical correspondence between natural transformations and equivariant maps of the corresponding standard fibres (see [11]), we find that the space of all linear natural transformations $TT_1^r T^* \to T^*T_1^r T^*$ over the identity of $T_1^r T^*$ forms a free finitely generated module over $C^\infty(\widetilde{B}_1, \dots, \widetilde{B}_r)$. This part of the proof is a technical matter, because we look for linear natural transformations only. $\square$

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