Differential Geometry and Its Applications
Proc. Conf. Opava (Czechoslovakia), August 24–28, 1992
Silesian University, Opava, 1993, 273–279

ON CERTAIN NATURAL TRANSFORMATIONS BETWEEN CONNECTIONS

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Abstract. We determine all natural transformations of $J^1(J^1(Y \to X) \to Y)$ into $J^1(J^1(Y \to X) \to X)$ over the identity of $J^1(Y \to X)$. Then we interpret the analytical result geometrically and correlate it with the geometry of connections on $Y \to X$.
Keywords. Natural operator, jet prolongation, connection.
MS classification. 53C05, 58A20.

1. Introduction

This work was motivated by some results from the higher-order dynamics on fibered manifolds with one-dimensional bases. In this situation, the second author has constructed the associated connection to a given dynamical connection on the jet prolongation of a fibered manifold [12], [13]. Both types of the connections have a dynamical meaning for regular variational problems. For related results on semisprays see e.g. [1].

In this paper we examine the possibility of such a construction for an arbitrary dimension of the base. We deduce that all natural transformations transforming connections on $J^1(Y \to X) \to Y$ into connections on $J^1(Y \to X) \to X$ form a one-parameter family. Our basic tool to prove this is the theory of natural bundles and operators [4], [6]. Then we present a geometrical interpretation of the analytical result and discuss its relations to the theory of natural prolongations of connections on the jet prolongation of a fibered manifold and also to some other topics [2], [3], [5], [9]. Throughout the paper we follow the notation and terminology of [7], [9], [11] and [4].

All manifolds and maps are assumed to be smooth.

2. Basic underlying structures and morphisms

In this paper we deal with a fibered manifold $\pi: Y \to X$ with local fibered coordinates (x^i, y^p), $i = 1, \dots, n = \dim X$, $p = 1, \dots, m = \dim Y - \dim X$. First we recall the structure of some jet prolongations of a manifold π.

The *first jet prolongation of* $\pi: Y \to X$ is the set J^1Y (also denoted by $J^1\pi$ or $J^1(Y \to X)$, if we intend to stress the bundle projection or the base, respectively) of all 1–jets of the local sections $\gamma: U \subset X \to Y$ of π. The coordinates (x^i, y^p) on Y induce

The authors would like to express their gratefulness to Prof. I. Kolář for his critical reading of the manuscript and valuable remarks.
This paper is in final form and no version of it will be submitted for publication elsewhere.

the additional coordinates y_i^p on J^1Y, defined by the formula $y_i^p(j_x^1\gamma) = \frac{\partial\gamma^p}{\partial x^i}(x)$. The manifold J^1Y can be considered as the total space of two different fibered manifolds; $\pi_1\colon J^1Y \to X$ and $\pi_{1,0}\colon J^1Y \to Y$. The latter one is an affine bundle modelled on the associated vector bundle $V_\pi Y \otimes T^*X \to Y$, where T^*X is the cotangent bundle of X and $V_\pi Y$ is the π-vertical subbundle of the tangent bundle TY. A *connection* on the fibered manifold $\pi\colon Y \to X$ is a section $\Gamma\colon Y \to J^1Y$. In the local coordinates, the equations of Γ are $y_i^p = \Gamma_i^p(x^i, y^q)$, where Γ_i^p are the *components* of Γ.

The set $J^1\pi_1 = J^1(J^1(Y \to X) \to X)$ of all 1–jets of local sections $\psi\colon U \subset X \to J^1Y$ is called the *second nonholonomic prolongation* of π (*repeated jets*). The induced coordinates on $J^1\pi_1$ are $(x^i, y^p, y_i^p, Y_i^p, Y_{ij}^p)$, where $Y_i^p(j_x^1\psi) = \frac{\partial\psi^p}{\partial x^i}(x)$ and $Y_{ij}^p(j_x^1\psi) = \frac{\partial\psi_i^p}{\partial x^j}(x)$. Then $(\pi_1)_{1,0}\colon J^1\pi_1 \to J^1\pi$ is an affine bundle, whose associated vector bundle is $V_{\pi_1}J^1\pi \otimes T^*X \to J^1\pi$. The coordinate expression of the bundle projection $(\pi_1)_{1,0}$ is $(x^i, y^p, y_i^p, Y_i^p, Y_{ij}^p) \mapsto (x^i, y^p, y_i^p)$. On the other hand, the jet prolongation $J^1(\pi_{1,0}; \mathrm{id}_X)$ of $\pi_{1,0}$ (considered as a fibered morphism over X) is another projection, locally defined by $J^1(\pi_{1,0}; \mathrm{id}_X)(j_x^1\psi) = j_x^1(\pi_{1,0} \circ \psi)$. Its expression is $(x^i, y^p, y_i^p, Y_i^p, Y_{ij}^p) \mapsto (x^i, y^p, Y_i^p)$.

The submanifold

$$\widehat{J}^2\pi = \{j_x^1\psi \in J^1\pi_1;\, J^1(\pi_{1,0}; \mathrm{id}_X)(j_x^1\psi) = (\pi_1)_{1,0}(j_x^1\psi)\}$$

of $J^1\pi_1$ is called the *second semiholonomic prolongation* of π. The restriction $\widehat{\pi}_{2,1} \equiv (\pi_1)_{1,0}\colon \widehat{J}^2\pi \to J^1\pi$ is an affine subbundle of $(\pi_1)_{1,0}$ with the associated vector bundle $V_\pi Y \otimes \overset{2}{\otimes} T^*X$. The sections of $\widehat{\pi}_{2,1}$ will be called *semiholonomic connections* on π_1.

The *second (holonomic) prolongation* of π is the set $J^2\pi$ of 2–jets of local sections of π. The local coordinates on $J^2\pi$ are $(x^i, y^p, y_i^p, y_{ij}^p)$, where $y_{ij}^p(j_x^2\gamma) = \frac{\partial^2\gamma^p}{\partial x^i \partial x^j}(x)$. Also $\pi_{2,1}\colon J^2\pi \to J^1\pi$ is an affine bundle with the associated vector bundle $V_\pi Y \otimes S^2T^*X \to J^1\pi$, where S^2T^*X is the second symmetric tensor power of the cotangent bundle T^*X. There is a canonical inclusion $i\colon J^2\pi \to J^1\pi_1$, $i(j_x^2\gamma) = j_x^1(j^1\gamma)$. For the sake of brevity we do not distinguish between $J^2\pi$ and $i(J^2\pi) \subset J^1\pi_1$. The sections of $\pi_{2,1}$ are called *holonomic conections* on π_1 or 2–*connections* on π.

Using the well-known exact sequence

$$0 \longrightarrow V_\pi Y \otimes T^*X \longrightarrow V_{\pi_1}J^1\pi \longrightarrow V_\pi Y \longrightarrow 0$$

it can be shown [7] that there is the canonical affine splitting

$$\widehat{J}^2\pi \cong J^2\pi \times_{J^1\pi} (V_\pi Y \otimes \Lambda^2T^*X). \tag{1}$$

While there is no canonical projection $J^1\pi_1 \to J^2\pi$, the splitting (1) gives rise to affine projections $s\colon \widehat{J}^2\pi \to J^2\pi$ and $r\colon \widehat{J}^2\pi \to V_\pi Y \otimes \Lambda^2T^*X$ expressing the symmetric or antisymmetric part of every fibered coordinate Y_{ij}^p, respectively.

Let $J^1(\Gamma; \mathrm{id}_X)\colon J^1\pi \to J^1\pi_1$ be the jet prolongation of a connection Γ on π considered as a morphism over X. The map $J^1(\Gamma; \mathrm{id}_X)$, which is used in the theory of the prolongations of connections, has the coordinate form

$$(x^i, y^p, y_i^p) \mapsto \left(x^i, y^p, \Gamma_i^p, y_i^p, \frac{\partial\Gamma_i^p}{\partial x^j} + \frac{\partial\Gamma_i^p}{\partial y^q}\, y_j^q\right).$$

It follows that $J^1(\Gamma; \mathrm{id}_X)$ is a section of the projection $J^1(\pi_{1,0}; \mathrm{id}_X)$ and the composition $J^1(\Gamma; \mathrm{id}_X) \circ \Gamma$ has the values in $\widehat{J}^2\pi$. The following equivalent conditions express the Frobenius theorem in terms of jets and connections :

(i) A connection Γ on π is integrable.

(ii) The composition $J^1(\Gamma; \mathrm{id}_X) \circ \Gamma$ has the values in $J^2\pi$.

(iii) For the curvature R_Γ of the connection Γ holds $R_\Gamma = -r \circ J^1(\Gamma; \mathrm{id}_X) \circ \Gamma = 0$.

Consider finally the set $J^1\pi_{1,0} = J^1(J^1(Y \to X) \to Y)$ of 1–jets of local sections of the bundle $\pi_{1,0} \colon J^1\pi \to Y$, i.e. of local connections on π. The induced coordinates on $J^1\pi_{1,0}$ are $(x^i, y^p, y^p_i, z^p_{ij}, z^p_{iq})$, where

$$z^p_{ij}(j^1_y\Gamma) = \frac{\partial \Gamma^p_i}{\partial x^j}(y), \quad z^p_{iq}(j^1_y\Gamma) = \frac{\partial \Gamma^p_i}{\partial y^q}(y).$$

Obviously, $(\pi_{1,0})_{1,0} \colon J^1\pi_{1,0} \to J^1\pi$ is an affine bundle with the associated vector bundle $V_{\pi_{1,0}} J^1\pi \otimes T^*Y \to J^1\pi$.

3. The main result

Let $\mathcal{FM}_{n,m}$ denote the category of fibered manifolds with n-dimensional bases and m-dimensional fibres and their local isomorphisms. Analogously to the concept of a classical natural bundle in the sense of Nijenhuis [10], a bundle functor on $\mathcal{FM}_{n,m}$ is a functor F transforming every $Y \in \mathrm{Ob}\,\mathcal{FM}_{n,m}$ into the fibered manifold $p_Y \colon FY \to Y$ over Y and every $(f \colon Y \to \overline{Y}) \in \mathrm{Mor}\,\mathcal{FM}_{n,m}$ into the fibered manifold morphism $Ff \colon FY \to F\overline{Y}$ over f [3]. Given two bundle functors F, G on $\mathcal{FM}_{n,m}$, by a *natural transformation* $T \colon F \to G$ we mean a system of base-preserving morphisms $T_Y \colon FY \to GY$, $Y \in \mathrm{Ob}\,\mathcal{FM}_{n,m}$, satisfying $Gf \circ T_Y = T_{\overline{Y}} \circ Ff$ for every $\mathcal{FM}_{n,m}$– morphism $f \colon Y \to \overline{Y}$.

First we prove the following technical result :

Lemma. *All natural transformations of $J^1\pi_{1,0}$ into $J^1\pi_1$ over the identity of $J^1\pi$ form a 1-parameter family*

$$\begin{aligned} Y^p_i &= y^p_i \\ Y^p_{ij} &= z^p_{ij} + z^p_{iq} y^q_j + a(z^p_{ij} - z^p_{ji} + z^p_{iq} y^q_j - z^p_{jq} y^q_i) \end{aligned} \tag{2}$$

with any $a \in \mathbb{R}$.

Proof. Let G^2_{n+m} be the group of all invertible 2–jets of diffeomorphisms of $\mathbb{R}^{n+m}$ with the source and the target zero and let $G^2_{n,m}$ be the subgroup of 2–jets of diffeomorphisms $\overline{x}^i = f^i(x)$, $\overline{y}^p = f^p(x, y)$ preserving the canonical fibration $\mathbb{R}^{n+m} \to \mathbb{R}^n$. Hence the coordinates on $G^2_{n,m}$ which correspond to the partial derivatives of f^i and f^p at the origin are

$$(a^i_j, a^i_{jk}, a^p_i, a^p_q, a^p_{ij}, a^p_{qi}, a^p_{qr}). \tag{3}$$

We shall denote by tilda the coordinates of the element inverse to (3) in $G^2_{n,m}$. According to the general theory [4], [6], the natural transformations of $J^1\pi_{1,0}$ into $J^1\pi_1$ are in the bijection with $G^2_{n,m}$- equivariant maps $S \to Z$ of the corresponding standard fibres. Hence we have to determine all $G^2_{n,m}$-maps of the form

$$\begin{aligned} Y^p_i &= g^p_i(y^p_i, z^p_{ij}, z^p_{iq}) \\ Y^p_{ij} &= g^p_{ij}(y^p_i, z^p_{ij}, z^p_{iq}). \end{aligned}$$

One evaluates easily the action of $G^2_{n,m}$ on S :

$$\begin{aligned}
\overline{y}^p_i &= a^p_q y^q_j \tilde{a}^j_i + a^p_j \tilde{a}^j_i \ , \\
\overline{z}^p_{ij} &= a^p_q z^q_{k\ell} \tilde{a}^k_i \tilde{a}^\ell_j + a^p_q z^q_{kr} \tilde{a}^r_j \tilde{a}^k_i + a^p_{q\ell} y^q_k \tilde{a}^k_i \tilde{a}^\ell_j + a^p_{qr} y^q_k \tilde{a}^r_j \tilde{a}^k_i + \\
&\quad + a^p_q y^q_k \tilde{a}^k_{ij} + a^p_{k\ell} \tilde{a}^k_i \tilde{a}^\ell_j + a^p_{kq} \tilde{a}^q_j \tilde{a}^k_i + a^p_k \tilde{a}^k_{ij} \ , \\
\overline{z}^p_{iq} &= a^p_r z^r_{js} \tilde{a}^s_q \tilde{a}^j_i + a^p_{rs} y^r_j \tilde{a}^s_q \tilde{a}^j_i + a^p_{rj} \tilde{a}^r_q \tilde{a}^j_i
\end{aligned}$$

and the action of $G^2_{n,m}$ on Z :

$$\begin{aligned}
\overline{Y}^p_i &= a^p_q Y^q_j \tilde{a}^j_i + a^p_j \tilde{a}^j_i \ , \\
\overline{Y}^p_{ij} &= a^p_q Y^q_{k\ell} \tilde{a}^k_i \tilde{a}^\ell_j + a^p_{qr} y^q_k Y^r_\ell \tilde{a}^k_i \tilde{a}^\ell_j + a^p_{qk} Y^q_\ell \tilde{a}^k_i \tilde{a}^\ell_j + a^p_{q\ell} y^q_k \tilde{a}^k_i \tilde{a}^\ell_j + \\
&\quad + a^p_q y^q_k \tilde{a}^k_{ij} + a^p_{k\ell} \tilde{a}^k_i \tilde{a}^\ell_j + a^p_k \tilde{a}^k_{ij}
\end{aligned}$$

(see also [3]). Consider first the map $g^p_i(y^p_i, z^p_{ij}, z^p_{iq})$. The equivariance with respect to the homotheties $\tilde{a}^i_j = k\,\delta^i_j$, $a^p_q = \delta^p_q$ gives a homogeneity condition

$$kg^p_i = g^p_i(ky^p_i, k^2 z^p_{ij}, kz^p_{iq}).$$

By the homogeneous function theorem [4], g^p_i are independent of z^p_{ij} and linear in y^p_i,z^p_{iq}. Analogously, the homotheties $a^i_j = \delta^i_j$, $a^p_q = k\,\delta^p_q$ yield

$$kg^p_i = g^p_i(ky^p_i, z^p_{iq}) \ ,$$

so that g^p_i are linear in y^p_i. Using the equivariance with respect to the subgroup $G \subset G^2_{n,m}$ with arbitrary a^i_j, a^p_q and with other a's vanishing, we find that g^p_i correspond to the $GL(n,\mathbb{R}) \times GL(m,\mathbb{R})$–invariant tensors. By the invariant tensor theorem [4] we get $g^p_i = a\,y^p_i$ with real parameter a. Finally, the full equivariance with respect to the subgroup $L \subset G^2_{n,m}$ characterized by $a^i_j = \delta^i_j$, $a^p_q = \delta^p_q$ reads $ay^p_i + a^p_i = a(y^p_i + a^p_i)$. Thus $a = 1$ and $g^p_i = y^p_i$, which corresponds to the first equation of (2). In the same way as in the case of g^p_i we find

$$g^p_{ij} = a\,z^p_{ij} + b\,z^p_{ji} + c\,y^p_i z^q_{qj} + d\,y^p_j z^q_{qi} + e\,y^q_i z^p_{qj} + f\,y^q_j z^p_{qi} \ ,$$

where $a, \dots, f$ are arbitrary real numbers. Then the equivariance with respect to the subgroup L implies $c = d = 0$, $b = 1 - a$, $e = 1 - a$, $f = a$, which corresponds to the second expression in (2). $\square$

Now we deal with the problem of how to transform the connections on $\pi_{1,0}$ into the connections on π_1. In other words, we transform the sections $J^1\pi \to J^1\pi_{1,0}$ into the sections $J^1\pi \to J^1\pi_1$. For this purpose, it is convenient to use the concept of a natural morphism operator [4]. If $Y_1 \to X$ and $Y_2 \to X$ are two fibered manifolds, denote by $C^\infty_X(Y_1, Y_2)$ the space of all base-preserving morphisms $Y_1 \to Y_2$. Given another pair $Z_1 \to X$ and $Z_2 \to X$ of fibered manifolds, a *morphism operator* $D\colon (Y_1, Y_2) \to (Z_1, Z_2)$ is a map $D\colon C^\infty_X(Y_1, Y_2) \to C^\infty_X(Z_1, Z_2)$. If we have four manifolds N_1, N_2, N_3, N_4, a map $\sigma\colon N_3 \to N_1$ and a subset $E \subset C^\infty(N_1, N_2)$, then an operator $A\colon E \to C^\infty(N_3, N_4)$ is called *σ-local* if the value $A\,s(x)$ depends only on the germ of s at $\sigma(x)$ for all $s \in E$, $x \in N_3$. Such an operator is said to be of order k, $0 \le k \le \infty$, if $j^k_{\sigma(x)} s_1 = j^k_{\sigma(x)} s_2$ implies $A\, s_1(x) = A\, s_2(x)$ for all $s_1, s_2 \in E$, $x \in N_3$. We say that A is *regular*, if it transformes every smoothly parametrized family of sections into a smoothly parametrized family. Let F_1, F_2, G_1, G_2 be bundle functors on $\mathcal{FM}_{n,m}$. A *natural operator* $D\colon (F_1, F_2) \to (G_1, G_2)$ is a system of regular operators $D_Y\colon (F_1Y, F_2Y) \to (G_1Y, G_2Y)$, $Y \in \operatorname{Ob}\mathcal{FM}_{n,m}$ such that for all $s_1 \in C^\infty_Y(F_1Y, F_2Y)$, $s_2 \in C^\infty_{\overline{Y}}(F_1\overline{Y}, F_2\overline{Y})$ and for every $\mathcal{FM}_{n,m}$– morphism $f\colon Y \to \overline{Y}$ the right hand diagram commutes whenever the left hand does :

$$\begin{array}{ccc} F_2Y & \xleftarrow{\ s_1\ } & F_1Y \\ {\scriptstyle F_2 f}\downarrow & & \downarrow{\scriptstyle F_1 f} \\ F_2\overline{Y} & \xleftarrow{\ s_2\ } & F_1\overline{Y} \end{array} \qquad\qquad \begin{array}{ccc} G_1Y & \xleftarrow{\ D_Y s_1\ } & G_2Y \\ {\scriptstyle G_1 f}\downarrow & & \downarrow{\scriptstyle G_2 f} \\ G_1\overline{Y} & \xleftarrow{\ D_{\overline{Y}} s_2\ } & G_2\overline{Y}. \end{array}$$

In our case, we have $F_1Y = G_1Y = J^1\pi$, $F_2Y = J^1\pi_{1,0}$, $G_2Y = J^1\pi_1$ and additionally we consider only the sections of $J^1\pi_{1,0} \to J^1\pi$ and $J^1\pi_1 \to J^1\pi$. From the theory of natural morphism operators it follows that all zero order natural operators transforming connections on $\pi_{1,0}$ into connections on π_1 are in bijection with the natural transformations $J^1\pi_{1,0} \to J^1\pi_1$ over $\mathrm{id}_{J^1\pi}$. So we can formulate our result in the following form :

Theorem. *All natural transformations transforming connections on $\pi_{1,0}\colon J^1Y \to Y$ into connections on $\pi_1\colon J^1Y \to X$ are of the form (2).*

4. Geometrical interpretations

The connection on $\pi_{1,0}$ is a section $\Xi\colon J^1\pi \to J^1\pi_{1,0}$. Then we may write $\Xi(z) = j^1_y\Gamma$, where $y = \pi_{1,0}(z)$ and $\Gamma\colon V \subset Y \to J^1\pi$ is a local connection on π.

The term $z^p_{ij} + z^p_{iq} y^q_j$ in (2) is a coordinate expression of the canonical map $J^1\pi_{1,0} \to J^1\pi_1$. This map has the form $j^1_y\Gamma \mapsto j^1_x(\Gamma\circ\gamma)$, where $\Gamma(y) = j^1_x\gamma$, $\gamma\colon U \subset X \to Y$. If γ is an *integral section* of Γ (i.e. $\Gamma\circ\gamma = j^1\gamma$ on U) then obviously $j^1_x(\Gamma\circ\gamma) = j^2_x\gamma \in J^2\pi$. This corresponds to the fact that $z^p_{ij} + z^p_{iq} y^q_j$ is the coordinate expression of the composition $J^1(\Gamma, \mathrm{id}_X) \circ \Gamma$.

Next we introduce the *formal curvature map*

$$R\colon J^1\pi_{1,0} \to V_\pi Y \otimes \Lambda^2 T^*X$$

by means of the formula

$$R^p_{ij} = \frac{1}{2}(z^p_{ij} - z^p_{ji} + z^p_{iq} y^q_j - z^p_{jq} y^q_i). \tag{4}$$

The well-known exact sequence $0 \to V_\pi Y \otimes T^*X \to V_{\pi_1} J^1\pi \to V_\pi Y \to 0$ implies $V_\pi Y \otimes \overset{2}{\otimes} T^*X \subset V_{\pi_1} J^1\pi \otimes T^*X$. Since $(\pi_1)_{1,0}\colon J^1\pi_1 \to J^1\pi$ is an affine bundle with the associated vector bundle $V_{\pi_1} J^1\pi \otimes T^*X \to J^1\pi$, the formal curvature (4) is a map $R\colon J^1\pi_{1,0} \to J^1\pi_1$.

Put

$$S^p_{ij} = \frac{1}{2}(z^p_{ij} + z^p_{ji} + z^p_{iq} y^q_j + z^p_{jq} y^q_i). \tag{5}$$

One evaluates easily that we can write (2) also in the form

$$
(6) \qquad \begin{aligned} Y^p_i &= y^p_i \\ Y^p_{ij} &= S^p_{ij} + b\, R^p_{ij}, \qquad b \in \mathbb{R}. \end{aligned}
$$

Then S^p_{ij} and R^p_{ij} express the symmetric or antisymmetric part of the term $z^p_{ij} + z^p_{iq} y^q_j$ in (2), respectively. This corresponds to the splitting (1) with the affine projections s, r. The antisymmetric curvature term R^p_{ij} in (2) and (6) is multiplied by an arbitrary real parameter. In this context we can mention the result of Kolář [3] : All natural operators transforming connections on $Y \to X$ into sections of $V_\pi Y \otimes \overset{2}{\otimes} T^*X$ are the real multiples of the curvature.

Let again Ξ be an arbitrary connection on $\pi_{1,0}$ and f any natural transformation from the Theorem. As a direct consequence of the Theorem we have :

1. All natural transformations transforming connections on $\pi_{1,0}$ into semiholonomic connections on π_1 are of the form $f(\Xi)$.

2. $s \circ f(\Xi)$ is the only natural transformation transforming connections on $\pi_{1,0}$ into holonomic connections on π_1.

This result creates a generalized and globalized version of that in [12], where Ξ is called a *dynamical connection* on $J^1\pi$ and $\Gamma^{(2)} = f(\Xi) = s \circ f(\Xi)$ is the associated *(semispray) connection* of order 2 on π with the same (dynamical) paths.

The one-parameter family $\widehat{\Gamma}^{(2)}_a := f(\Xi)$ of semiholonomic connections induces a one-parameter system of morphisms

$$
R_a := r \circ f(\Xi) \colon J^1\pi \to V_\pi Y \otimes \Lambda^2 T^*X.
$$

Then R_a is the *torsion* of the corresponding semiholonomic connection $\widehat{\Gamma}^{(2)}_a$ and $R_a = 0$ if and only if $\widehat{\Gamma}^{(2)}_a$ is holonomic. This corresponds to Saunders [11].

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Received 25 August 1992