Differential Geometry and Its Applications
Proc. Conf. Opava (Czechoslovakia), August 24–28, 1992
Silesian University, Opava, 1993, 3–12

Openess and Density Theorems of Transversality in Manifolds with Corners *

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Abstract

In this paper we deal with infinitesimal transversality, transversality, boundary-transversality and weak-boundary-transversality defined on manifolds with corners. We prove openess theorems about these types of transversality. The main theorem concerns the residuality of the jet-transversality (boundary-transversality and weak-boundary-transversality).

1991 *Mathematics Subject Classification. Primary:* 58C25, 58D15. *Secondary:* 58C27.
Key words and phrases: Transversality, boundary-transversality, weak-boundary-transversality, infinitesimal transversality, Abraham-Quinn parametrized theorems of the density of the transversality, openess of transversality, Thom Density theorem of Transversality.

1 Introduction

We are interested in studying transversality of maps to a submanifold, especially openess and density of transversality. We deal with infinitesimal transversality [1], [2], transversality [8], boundary-transversality [3], and weak-boundary-transversality [4]. First we prove that

$$\begin{aligned}\{f \in C^p(X,X') \ / \ &\text{for every } x \in X, f(x) \notin \overline{Y_o} \text{ or}\\ &f(x) \in \overline{Y_o} \text{ and } f \text{ is infinitesimally transversal to } Y \text{ at } x\}\end{aligned}$$

is an open set of $C^p(X,X')$ with the Whitney topology, where X and X' are general manifolds (X, X' can be infinite dimensional manifolds). From this result it follows that $\{g \in C^p(X,X') \ / \text{ for every } x \in X_i^h \text{ with } g(x) \in \overline{Y_o},\ g_{|B_hX} \pitchfork_x Y\}$ is an open set of $C^p(X,X')$ with the Whitney topology, where X is finite dimensional, $B_hX = \bigcup_{i\in I\!N} X_i^h$ is a decomposition of $B_k(X)$ by means of a stratification of compact submanifolds and $\overline{Y_o} \subseteq Y$. Thus if $\partial X = \emptyset$, we obtain that G is an open set of $C^p(X,X')$ with the Whitney topology, where

$$\begin{aligned}G &= \{g \in C^p(X,X') \ / \text{ for every } x \in X,\ g(x) \notin \overline{Y_o} \text{ or } g(x) \in \overline{Y_o}, \text{ and } g \pitchfork_x^* Y\} =\\ &= \{g \in C^p(X,X') \ / \text{ for every } x \in X,\ g(x) \notin \overline{Y_o} \text{ or } g(x) \in \overline{Y_o}, \text{ and } g \pitchfork_x^\partial Y\} =\\ &= \{g \in C^p(X,X') \ / \text{ for every } x \in X,\ g(x) \notin \overline{Y_o} \text{ or } g(x) \in \overline{Y_o}, \text{ and } g \pitchfork_x Y\}.\end{aligned}$$

*Supported by DGICYT-SPAIN, Proyecto PB89-0122 and Consejería de Educación del Gobierno de Canarias-SPAIN
This paper is in final form and no version of it will be submitted for publication elsewhere.

Finally in the main theorems, 3.2 and 3.3, we prove the residuality of the sets $\{g \in C^p(X,X') \ / \ j^r g \pitchfork Y\}$, $\{g \in C^p(X,X') \ / \ j^r g \pitchfork^* Y\}$ and $\{g \in C^p(X,X') \ / \ j^r g \pitchfork^\partial Y\}$, with the Whitney topology, which generalize Thom's theorem of the residuality of the jet-transversality [11].

2 Prerequisites

Throughout this paper manifolds will be Banach manifolds and they may have boundary if otherwise is not specified. Terminology and notations can be found in [8], [9] and [7].

Let X be a C^p-manifold and Y a C^p-submanifold, we say Y is a totally neat submanifold if $ind_Y(x) = ind_X(x)$ for every $x \in Y$. If only $\partial Y = Y \cap \partial X$, one says that Y is a neat submanifold.

Let $f : X \longrightarrow X'$ be a C^p-map and $x_0 \in X$. The map f is called a submersion at x_0 if there are an open neighbourhood $V^{f(x_0)}$ of $f(x_0)$ in X' and a C^p-map $s : V^{f(x_0)} \longrightarrow X$ such that $s(f(x_0)) = x_0$ and $(f \circ s)(y) = y$ for all $y \in V^{f(x_0)}$. We take this definition in order to have the following universal property: *If f is a surjective submersion and g is a map, then $g \circ f$ is a C^p-map if and only if g is a C^p-map.* Nevertheless if f preserves the boundary at x, this definition is equivalent to the classical one: *$T_x f$ is surjective and its kernel admits a topological supplement in $T_x X$*. Here, of course, f preserves the boundary at x means that there exists an open neighbourhood V^x of x in X such that $f(V^x \cap \partial X) \subseteq \partial X'$. In any case the definition of submersion at x always implies the classical one.

The classical notion of transversality or infinitesimal transversality can be found in [1], [2], and the notion of transversality (local formulation) can be found in [8]. Next we explain the boundary and weak-boundary transversality ([3], [4]).

Definition 2.1 *Let X, X' be C^p-manifolds, $f : X \longrightarrow X'$ a C^p-map and Y' a C^p-submanifold of X'. The map f is called boundary-transversal (respectively weak-boundary-transversal) to Y' at $x \in X$, and it will be denoted by $f \pitchfork^\partial_x Y'$ (respectively $f \pitchfork^*_x Y'$), if $f(x) \notin Y'$ or $f(x) \in Y'$ and there are a chart $c' = (U', \varphi', (E', \Lambda'))$ of X' adapted to Y' at $f(x)$ by means of (E'_1, Λ'_1), a topological supplement E'_2 of E'_1 in E', a finite linearly independent system Λ'_2 of $L(E'_2, I\!R)$ and an open neighbourhood U of x in X, such that $f(U) \subseteq U'$,*

$$(p'_2 \circ (\theta')^{-1} \circ \varphi' \circ f)((U) \subseteq (E'_2)^+_{\Lambda'_2}, \quad p'_2(\partial(E'_1 \times E'_2)^+_{\Lambda' \circ \theta'} \cap ((\theta')^{-1} \circ \varphi' \circ f)(\partial U)) \subseteq \partial(E'_2)^+_{\Lambda'_2}$$

and the C^p-map $h : U \longrightarrow (E'_2)^+_{\Lambda'_2}$, defined by $h(y) = (p'_2 \circ (\theta')^{-1} \circ \varphi' \circ f_{|U})(y)$ for every $y \in U$ is a submersion at x, (respectively $f(U) \subseteq U'$, $(p'_2 \circ (\theta')^{-1} \circ \varphi' \circ f)(U) \subseteq (E'_2)^+_{\Lambda'_2}$ and the C^p-map $h : U \longrightarrow (E'_2)^+_{\Lambda'_2}$, defined by $h(y) = (p'_2 \circ (\theta')^{-1} \circ \varphi' \circ f_{|U})(y)$ for every $y \in U$ is a submersion at x) being $\theta' : E'_1 \times E'_2 \longrightarrow E'$ the linear homeomorphism defined by $\theta'(a', b') = a' + b'$, p'_2 the second projection from $E'_1 \times E'_2$ onto E'_2 and $\Lambda' \circ \theta' = \{\lambda' \circ \theta' \ / \ \lambda' \in \Lambda'\}$.

If A is a subset of X and $f \pitchfork^\partial_x Y'$ ($f \pitchfork^*_x Y'$) for every $x \in A$, we will say that f is boundary-transversal (weak-boundary-transversal) to Y' along A and it will be denoted by $f \pitchfork^\partial_A Y'$ ($f \pitchfork^*_A Y'$). If $A = X$, one says f is boundary-transversal (weak-boundary-transversal) to Y' and denotes by $f \pitchfork^\partial Y'$ ($f \pitchfork^* Y'$).

It is obvious that transversality [8] and boundary-transversality involve the weak-boundary-transversality.

Proposition 2.1 *[7] Let $f : X \longrightarrow X'$ be a C^p-map, Y' a C^p-submanifold of X' and $x \in f^{-1}(Y')$. Suppose that $f \pitchfork_x Y'$, ($f \pitchfork_x^{\partial} Y'$ or $f \pitchfork_x^{*} Y'$) then it holds:*

(a) $T_{f(x)}X' = im(T_x f) + (T_{f(x)}j')(T_{f(x)}Y')$ and

(b) $T_x f)^{-1}[(T_{f(x)}j')(T_{f(x)}Y')]$ admits a topological supplement in $T_x X$
where $j' : Y' \hookrightarrow X'$ is the inclusion map.

If *(a)* and *(b)* of the preceding proposition hold, we shall say that f is *infinitesimally transversal* to Y' at x or briefly *f is inf-t. to Y' at x.*

On the other hand, if $x \in int(X)$, then $f \pitchfork_x Y' \;\Leftrightarrow\; f \pitchfork_x^{*} Y' \;\Leftrightarrow\; f \pitchfork_x^{\partial} Y'$. Moreover if $x \in f^{-1}(Y') \cap int(X)$, one has $f \pitchfork_x^{*} Y'$ if and only if f is infinitesimally transversal to Y' at x.

If $ind(x) = k$, then $f_{|B_k X} : B_k X \longrightarrow X'$ is infinitesimally transversal to Y' at x, (that is $f_{|B_k X} \pitchfork_x Y'$) if and only if $f \pitchfork_x Y'$ *(see [8])*.

Proposition 2.2 *Let $f : X \to X'$ be a C^p-map, Y a C^p-submanifold of X' and $x \in f^{-1}(Y)$. Then, if f preserves the boundary at x and $(T_{f(x)}j')(T_{f(x)}Y) \subseteq (T_{f(x)}X')^i$, the following statements are equivalent:*

$$a)\ f \text{ is inf-t. to } Y \text{ at } x, \qquad b)\ f \pitchfork_x^{\partial} Y, \qquad c)\ f \pitchfork_x^{*} Y.$$

Let M, X and X' be C^p-manifolds and ϱ a map from M into $C^p(X, X')$. One says ϱ is a C^p-representation if the map $e_\varrho : M \times X \longrightarrow X'$, defined by $e_\varrho(m, x) = \varrho(m)(x)$, is a C^p-map. In [4] we give a parametrized theorem of the density of the boundary-transversality.

Theorem 2.1 *Let M, X and X' be C^p-manifolds, $p \geq 1$, $\varrho : M \to C^p(X, X')$ a C^p-representation and W a C^p-submanifold of X' which is a neat submanifold or is without boundary. Suppose that M is a T_3 second countable space, X is a Hausdorff second countable space, $dim_x(X) = n < +\infty$ for every $x \in X$, $codim_y(W) = q < +\infty$ for every $y \in W$, $p - 1 \geq n - q$, e_ϱ preserves the boundary at every $z \in (e_\varrho)^{-1}(W)$ and $e_\varrho \pitchfork^{\partial} W$. Then $\{m \in M \mid \varrho(m) \pitchfork^{\partial} W\}$ is a residual set in M and therefore dense in M.*

Let X, X' be C^p-manifolds, $p \geq 1$, k an integer, $0 \leq k \leq p$, and $J^k(X, X')$ the set of k-jets from X to X'. Then there is a unique topology $T^{(k)}$ on $J^k(X, X')$ such that for any two charts $c = (U, \varphi, (E, \Lambda))$ of X and $c' = (U', \varphi', (E', \Lambda'))$ of X', the set $J^k(U, U')$ is open in $T^{(k)}$ and the map:

$$\pi_c^{c'} : J^k(U, U') \longrightarrow \varphi(U) \times \varphi'(U') \times L(E, E') \times \ldots \times L_s^k(E, E')$$

defined by $\pi_c^{c'}(j_x^k f) = (\varphi(x), \varphi'(f(x)), D(\varphi' \circ f \circ \varphi^{-1})(\varphi(x)), \ldots, D^k(\varphi' \circ f \circ \varphi^{-1})(\varphi(x))$ is a homeomorphism onto $im(\pi_c^{c'})$. Moreover if $\partial X' = \emptyset$, there exists a unique differentiable structure $[A]$ of class $p - k$ on $J^k(X, X')$ such that for every chart $c = (U, \varphi, (E, \Lambda))$ of X and for every chart $c' = (U', \varphi', E')$ of X' the tuple $d_c^{c'} = (J^k(U, U'), \pi_c^{c'}, (E \times E' \times L(E, E') \times \ldots \times L_s^k(E, E'), \Lambda \circ p_1))$ is a chart of $(J^k(X, X'), [A])$.

The topology of this manifold is the coherent topology $T^{(k)}$, the maps $s : J^k(X,X') \to X$ and $b : J^k(X,X') \to X'$ defined by $s(j^k_x f) = x$ and $b(j^k_x f) = f(x)$ are surjective C^{p-k}-submersions and

$$\partial^q J^k(X,X') = s^{-1}(\partial^q X), \;\; B_q J^k(X,X') = s^{-1}(B_q X), \;\; q \geq 0.$$

Definition 2.2 *Let X, X' be C^p-manifolds, $p \in I\!N \cup \{\infty\}$, k an integer with $0 \leq k \leq p$ and T_s the graph topology on $C(X, J^k(X,X'))$ (see [8]). Consider the map $j^k : C^p(X,X') \to C(X, J^k(X,X'))$ defined by $(j^k(f))(x) = j^k_x f$. Then the inverse image topology $(j^k)^{-1}(T_s)$, will be called Whitney topology of order k on $C^p(X,X')$ and will be denoted by $T_W(p,k)$.*

Then we have $B_W(p,k) = \{M(G) \; / \; G$ is an open set of $J^k(X,X')$ and $M(G) \neq \emptyset\}$ is a basis of $T_W(p,k)$, where $M(G) = \{f \in C^p(X,X') \; / \; (j^k f)(X) \subseteq G\}$.

3 Openess and Density of Transversality

In [8] one makes a decomposition of a manifold without boundary by means of a stratification of compact submanifolds.

Proposition 3.1 *Let X be a second countable Hausdorff non-compact C^p-manifold, $p \in I\!N \cup \{\infty\}$, with $\partial X = \emptyset$. Suppose that $dim_x(X) = m \in I\!N$ for every $x \in X$. Then $X = \bigcup_{i \in I\!N} X_i$, where X_i is a compact C^p-submanifold of X with $\partial X_i \neq \emptyset$, $\partial^2 X_i = \emptyset$, $int(X_i)$ is an open set of X, $X_i \subseteq X^\circ_{i+1}$ for every $i \in I\!N$. Moreover $X^\circ_i = int(X_i)$ and $dim(X_i) = m$ for every $i \in I\!N$, where X°_i is the topological interior of X_i in X.*

Corollary 3.1 *Let X be a second countable Hausdorff C^p-manifold, $p \in I\!N \cup \{\infty\}$, with $\partial X = \emptyset$. Suppose that $dim_x(X) = m \in I\!N \cup \{0\}$ for every $x \in X$. Then $X = \bigcup_{i \in I\!N} X_i$, where X_i is a compact C^p-submanifold of X with $\partial^2 X_i = \emptyset$, $X_i \subseteq X^\circ_{i+1}$, $int(X_i)$ is an open set of X and $dim(X_i) = m$ for every $i \in I\!N$.*

Proof: If X is compact, we take $X_i = X$ for every $i \in I\!N$. If $m \geq 1$ and X is not compact, it follows from the proposition 3.1. If $m = 0$ and X is not compact, then X is not finite, $X = \bigcup_{n \in I\!N} \{x_n\}$ and we take $X_n = \{x_k \; / \; 1 \leq k \leq n\}$ for every $n \in I\!N$. □

Lemma 3.1 *Let X, X' be C^p-manifolds, $p \in I\!N \cup \{\infty\}$, $f : X \longrightarrow X'$ a C^p-map, Y' a C^p-submanifold of X', $x \in f^{-1}(Y')$, $c = (U, \varphi, (E, \Lambda))$ a chart of X centred at x, $c' = (U', \varphi', (E', \Lambda'))$ a chart of X' adapted to Y' at $f(x)$ by means of (E'_1, Λ'_1) with $f(U) \subseteq U'$, E'_2 a topological supplement of E'_1 in E' and h the map defined by:*

$$h : \varphi(U) \xrightarrow{\varphi^{-1}} U \xrightarrow{f_{|U}} U' \xrightarrow{\varphi'} \varphi'(U') \xrightarrow{j''} E' \xrightarrow{(\theta')^{-1}} E'_1 \times E'_2 \xrightarrow{p'_2} E'_2.$$

If $y \in U \cap f^{-1}(Y')$, then we have that $Dh(\varphi(y)) : E \longrightarrow E'_2$ is surjective and its kernel admits topological supplement in E if and only if f is inf-t. to Y at y.

Theorem 3.1 *Let X, X' be C^p-manifolds, $p \in I\!N \cup \{\infty\}$, Y a C^p-submanifold of X', Y_o a subset of Y such that $Y_o \neq \emptyset$, $\overline{Y_o} \subseteq Y$ and $r \in I\!N$ with $1 \leq r \leq p$. Then*

$$G = \{f \in C^p(X,X') \; / \; \textit{for every } x \in X, \, f(x) \notin \overline{Y_o} \textit{ or } f(x) \in \overline{Y_o} \textit{ and } f \textit{ is inf-t. to } Y \textit{ at } x\}$$

is an open set of $(C^p(X,X'), T_W(p,r))$.

Proof: Let us consider

$$U^r = \{j^r_x f \in J^r(X,X') \;/\; f(x) \notin \overline{Y_o} \text{ or } f(x) \in \overline{Y_o} \text{ and } f \text{ is inf-t. to } Y \text{ at } x\}.$$

Let $\{j^r_{x_n} f_n\}_{n\in I\!N}$ be a sequence in $C = J^r(X,X') - U^r$ which converges to $j^r_{x_o} f_o \in J^r(X,X')$ with the topology $T^{(r)}$. Consequently $\{x_n\}_{n\in I\!N}$ converges to x_o and $\{f_n(x_n)\}_{n\in I\!N}$ converges to $x'_o = f_o(x_o)$. For every $n \in I\!N$, $f_n(x_n) = x'_n \in \overline{Y_o}$ and f_n is not infinitesimally transversal to Y at x_n. Then $x'_o \in \overline{Y_o}$.

Now we consider a chart $c = (U, \varphi, (E, \Lambda))$ of X centred at x_o with $U \subseteq dom(f_o)$ and a chart $c' = (U', \varphi', (E', \Lambda'))$ of X' adapted to Y at $x'_o \in Y$ by means of (E', Λ') such that $f_o(U) \subseteq U'$. Let us consider the map h_o defined by

$$h_o : \varphi(U) \xrightarrow{\varphi^{-1}} U \xrightarrow{f_{o|U}} U' \xrightarrow{\varphi'} \varphi'(U') \xrightarrow{j''} E' \xrightarrow{\theta^{-1}} E'_1 \times E'_2 \xrightarrow{p_2} E'_2,$$

where E'_2 is a topological supplement of E'_1 in E'. There exists $n_o \in I\!N$ such that for every $n \geq n_o$, there exists an open set V_n of $\varphi(U) \subseteq E^+_\Lambda$ with $\varphi(x_n) \in V_n$, $\varphi^{-1}(V_n) \subseteq dom(f_n)$ and $f_n(\varphi^{-1}(V_n)) \subseteq U'$. Then we can define the map h_n, $n \geq n_o$, as it follows:

$$h_n : V_n \xrightarrow{\varphi^{-1}} \varphi^{-1}(V_n) \xrightarrow{f_n} U' \xrightarrow{\varphi'} \varphi'(U') \xrightarrow{j''} E' \xrightarrow{\theta^{-1}} E'_1 \times E'_2 \xrightarrow{p_2} E'_2,$$

We have $\{Dh_n(\varphi(x_n))\}_{n\in I\!N}$ converges to $Dh_o(\varphi(x_o))$ in $L(E, E'_2)$. Suppose that f_o is infinitesimally transversal to Y at x_o. Then $Dh_o(0) : E \to E'_2$ is surjective and $Ker(Dh_o(0))$ admits a topological supplement in E. Therefore $Dh_o(0) \in H = \{l \in L(E, E'_2) \;/\; l \text{ is surjective and } Ker(l) \text{ admits a topological supplement in } E\}$ which is open in $L(E, E'_2)$. Thus there is $m_o \geq n_o$ such that $Dh_n(\varphi(x_n)) \in H$ for $n \geq m_o$. By the lemma 3.1, for every $n \geq m_o \geq n_o$, f_n is infinitesimally transversal to Y at x_n, which is a contradiction. Then C is closed and $M(U^r)$ is open in $(C^p(X,X'), T_W(p,r))$. Finally $M(U^r) = G$. □

Corollary 3.2 *Let X, X' be Hausdorff C^p-manifolds, $p \in I\!N \cup \{\infty\}$, where X is a second countable manifold. Suppose that $dim_x(X) = m \in I\!N$ for every $x \in X$. Consider an integer r with $1 \leq r \leq p$, a C^p-submanifold Y of X', a subset Y_o of Y such that $Y_o \neq \emptyset$, $\overline{Y_o} \subseteq Y$ and $B_h X = \bigcup_{i\in I\!N} X^h_i$ for every $h \in \{0,\ldots,m\}$ as in the corollary 3.1 (notice that $dim(B_h X) = m - h$). Then for every $i \in I\!N$ and every $h \in \{0,\ldots,m\}$,*

$$G^h_i = \{g \in C^p(X,X') \;/\; \textit{for every } x \in X^h_i \textit{ with } g(x) \in \overline{Y_o},\ g_{|B_h X} \pitchfork_x Y\}$$

is an open set of $(C^p(X,X'), T_W(p,r))$.

Proof: It is a consequence of the theorem 3.1, (note that $T_x X^h_i = T_x B_h X$) and of (since $j : X^h_i \hookrightarrow X$ is a proper map) the continuity of the restriction map $j^* : C^p(X,X') \to C^p(X^h_i, X')$ with the topology $T_W(p,r)$. □

Corollary 3.3 *In the hypotheses of the theorem 3.1, if we suppose that $\partial X \cap f^{-1}(\overline{Y_o}) = \emptyset$, then*

$$\begin{aligned} G &= \{f \in C^p(X,X') \;/\; \textit{for every } x \in X, f(x) \notin \overline{Y_o} \textit{ or } f(x) \in \overline{Y_o} \textit{ and } f \pitchfork_x Y\} = \\ &= \{f \in C^p(X,X') \;/\; \textit{for every } x \in X, f(x) \notin \overline{Y_o} \textit{ or } f(x) \in \overline{Y_o} \textit{ and } f \pitchfork^*_x Y\} = \\ &= \{f \in C^p(X,X') \;/\; \textit{for every } x \in X, f(x) \notin \overline{Y_o} \textit{ or } f(x) \in \overline{Y_o} \textit{ and } f \pitchfork^\partial_x Y\}, \end{aligned}$$

is open of $(C^p(X,X'), T_W(p,r))$.

Proposition 3.2 *In the conditions of the theorem 3.1, if G is the set defined in that theorem and we suppose that for every $y \in \overline{Y_o}$, $(T_y j)(T_y Y) \subseteq (T_y X')^i$, where $j : Y \hookrightarrow X'$ is the inclusion map, then we have:*

1. *If $C^p_\partial(X, X') = \{f \in C^p(X, X') \;/\; f$ preserves the boundary$\}$, then $G \cap C^p_\partial(X, X')$ is an open set of $(C^p_\partial(X, X'), T_W(p, r)_{|C^p_\partial(X,X')})$ and*

$$G \cap C^p_\partial(X, X') = \{f \in C^p_\partial(X, X') \;/\; \text{for every } x \in X, f(x) \notin \overline{Y_o} \text{ or } f(x) \in \overline{Y_o} \text{ and } f \pitchfork^\partial_x Y\}.$$

2. *If $C^p_{\partial,\overline{Y_o}}(X, X') = \{f \in C^p(X, X') \,/\, f$ preserves the boundary at every $x \in f^{-1}(\overline{Y_o})\}$, then $G_1 = G \cap C^p_{\partial,\overline{Y_o}}(X, X')$ is an open set of $(C^p_{\partial,\overline{Y_o}}(X, X'), T_W(p, r)_{|C^p_{\partial,\overline{Y_o}}(X,X')})$ and*

$$\begin{aligned} G_1 &= \{f \in C^p_{\partial,\overline{Y_o}}(X, X') \;/\; \text{for every } x \in X, f(x) \notin \overline{Y_o} \text{ or } f(x) \in \overline{Y_o} \text{ and } f \pitchfork^\partial_x Y\} \\ &= \{f \in C^p_{\partial,\overline{Y_o}}(X, X') \;/\; \text{for every } x \in X, f(x) \notin \overline{Y_o} \text{ or } f(x) \in \overline{Y_o} \text{ and } f \pitchfork^*_x Y\} \end{aligned}$$

Proposition 3.3 *Let X, X' be C^p-manifolds such that $\partial X' = \emptyset$, $p \in I\!N \cup \{\infty\}$ and $r \in Z\!\!Z$ with $0 \le r < p$. Let Y be a C^{p-r}-submanifold of $J^r(X, X')$, X_o a C^p-submanifold of X such that the inclusion $j : X_o \hookrightarrow X$ is proper and $t \in Z\!\!Z$ with $1 \le t \le p - r$. Suppose that $Y = \bigcup_{m \in I\!N} Y_m$ with $\overline{Y_m} \neq \emptyset$, $\overline{Y_m} \subseteq Y$ for every $m \in I\!N$. Then:*

1. *If X and X' are Hausdorff and X is paracompact,*

$$(T^{X_o}_m)^r = \{g \in C^p(X, X') \;/\; \text{for every } x \in X_o \text{ with } j^r_x g \in \overline{Y_m}, (j^r g)_{|X_o} \text{ is inf-t. to } Y \text{ at } x\}$$

is an open set of $(C^p(X, X'), T_W(p, t + r))$, for every $m \in I\!N$.

2. *$T^r_m = \{g \in C^p(X, X') \;/\;$ for every $x \in X$ with $j^r_x g \in \overline{Y_m}, j^r g$ is inf-t. to Y at $x\}$ is an open set of $(C^p(X, X'), T_W(p, t + r))$, for every $m \in I\!N$.*

3. *$\{g \in C^p(X, X') \,/\, j^r g$ is inf-t. to Y at x, for every $x \in X$ with $j^r_x g \in Y\}$ $= \bigcap_{m \in I\!N} T^r_m$.*

Proof: *1.* For every $m \in I\!N$ we have, by the theorem 3.1,

$$H_m = \{h \in C^{p-r}(X_o, J^r(X, X')) \;/\; \text{for every } x \in X_o, h(x) \notin \overline{Y_m} \text{ or } h(x) \in \overline{Y_m} \text{ and } h \text{ is inf-t. to } Y \text{ at } x\}$$

is an open set of $(C^{p-r}(X_o, J^r(X, X')), T_W(p - r, t))$. Since the map $j : X_o \hookrightarrow X$ is proper, the restriction map $j^* : C^{p-r}(X, J^r(X, X')) \longrightarrow C^{p-r}(X_o, J^r(X, X'))$ is continuous with $T_W(p - r, t)$. So $(j^*)^{-1}(H_m)$ is open in $(C^{p-r}(X, J^r(X, X')), T_W(p - r, t))$. Finally $(T^{X_o}_m)^r = (j^r)^{-1}(j^*)^{-1}(H_m)$ and $j^r : (C^p(X, X'), T_W(p, t + r)) \to (C^{p-r}(X, J^r(X, X')), T_W(p - r, t))$ defined by $j^r(f)(x) = j^r_x f$ is continuous.

2 and *3* are easy to be checked. □

Corollary 3.4 *Let X, X' be a Hausdorff C^p-manifolds, $p \in I\!N \cup \{\infty\}$ such that for every $x \in X$, $dim_x(X) = n \in I\!N$. Suppose that $\partial X' = \emptyset$ and X is second countable. Let Y be a C^{p-r}-submanifold of $J^r(X, X')$ and $t \in Z\!\!Z$ with $1 \leq t \leq p - r$. If $Y = \bigcup_{m \in I\!N} Y_m$ with $\overline{Y_m} \neq \emptyset$, $\overline{Y_m} \subseteq Y$ for every $m \in I\!N$ and $B_k X = \bigcup_{i \in I\!N} X_i^k$ for every $k \in \{0, \ldots, n\}$, where X_i^k is a compact C^p-submanifold of $B_k X$ with $\partial^2 X_i^k = \emptyset$, $int(X_i^k)$ is an open set of $B_k X$ and $dim(X_i^k) = n - k$ for every $i \in I\!N$, then for every $m \in I\!N$, $i \in I\!N$ and $k \in \{0, \ldots, n\}$,*

$$T(m,k,i) = \{g \in C^p(X, X') \ / \ \text{for every } x \in X_i^k \text{ with } j_x^r g \in \overline{Y_m}, (j^r g)_{|B_k X} \pitchfork_x Y\}$$

is an open set of $(C^p(X, X'), T_W(p, t + r))$ and $\{g \in C^p(X, X') \ / \ j^r g \pitchfork Y\} = \bigcap_{m,k,i} T(m, k, i)$.

Proposition 3.4 *With the same hypotheses and notation of the proposition 3.3, if for every $y \in Y$, $(T_y j)(T_y Y) \subseteq (T_y J^r(X, X'))^i$ then*

1. *$T_m^r = \{f \in C^p(X, X') \ / \ \text{for every } x \in X, j_x^r f \notin \overline{Y_m} \text{ or } j_x^r f \in \overline{Y_m} \text{ and } j^r f \pitchfork_x^\partial Y\} =$*
   $= \{f \in C^p(X, X' \ / \ \text{for every } x \in X, j_x^r f \notin \overline{Y_m} \text{ or } j_x^r f \in \overline{Y_m} \text{ and } j^r f \pitchfork_x^* Y\}$.
2. *$\bigcap_{m \in I\!N} T_m^r = \{f \in C^p(X, X') \ / \ j^r f \pitchfork^\partial Y\} = \{f \in C^p(X, X') \ / \ j^r f \pitchfork^* Y\}$.*
3. *$\{g \in C^p(X, X') \ / \ j^r g \pitchfork Y\} \subseteq \bigcap_{m \in I\!N} T_m^r$.*

Theorem 3.2 *Let X, X' be Hausdorff second countable C^p-manifolds, $p \in I\!N \cup \{\infty\}$, and r an integer, $0 \leq r < p$. Suppose that $\partial X' = \emptyset$, $dim_x(X) = n \in I\!N$ for every $x \in X$ and $dim_{x'}(X') < +\infty$ for every $x' \in X'$. Let Y be a C^{p-r}-submanifold of $J^r(X, X')$ without boundary and $codim_y Y = q \in I\!N$ for every $y \in Y$, verifying $p - r - 1 \geq n - q$ and $t \in Z\!\!Z$ with $1 \leq t \leq p - r$. Then $(T^\partial)_Y^r = \{g \in C^p(X, X') \ / \ j^r g \pitchfork^\partial Y\}$ is a residual, hence dense, set in $(C^p(X, X'), T_W(p, r + t))$. Moreover, $(T^\partial)_Y^r = (T^*)_Y^r = \{g \in C^p(X, X') \ / \ j^r g \pitchfork^* Y\} \supseteq T_Y^r = \{g \in C^p(X, X') \ / \ j^r g \pitchfork Y\}$.*

Proof: Since Y is a C^{p-r}-submanifold of $J^r(X, X')$, $p - r > 0$, then $Y = G \cap C$ where G is open and C is closed in $J^r(X, X')$ with the topology $T^{(r)}$. Moreover $J^r(X, X')$ is a Hausdorff, second countable, regular manifold.

For every $y \in Y$ we take charts $c_y = (U_y, \varphi_y, (I\!R^n, \Lambda_y))$, $c'_y = (U'_y, \varphi'_y, I\!R^{n'_y})$ of X and X' centred at $s(y)$ and $b(y)$ respectively, such that $\overline{U_y}$, $\overline{U'_y}$ are compact sets and $\varphi_y(U_y)$, $\varphi'_y(U'_y)$ are bounded sets. Let V_y be an open neighbourhood of y in $J^r(X, X')$ such that $\overline{V_y} \subseteq G$, $\overline{V_y}$ is compact and $(s, b)(\overline{V_y}) \subseteq U_y \times U'_y$. Then $V_y \cap Y = Y_y$ is an open set of Y, $y \in Y_y$, $\overline{Y_y} \subseteq \overline{V_y} \cap C \subseteq G \cap C = Y$, $\overline{Y_y}$ is compact and $(s, b)(\overline{Y_y}) \subseteq U_y \times U'_y$, where $\overline{Y_y}$ is the adherence of Y_y in $J^r(X, X')$. Then $Y = \bigcup_{m \in I\!N} Y_{y_m}$ and, by the proposition 3.4 (notice that, for every $y \in Y$, $T_y j(T_y Y) \subseteq (T_y J^r(X, X'))^i$ being $j : Y \hookrightarrow X$ the inclusion map), for every $m \in I\!N$,

$$T_m^r = \{g \in C^p(X, X') \ / \ \text{for every } x \in X \text{ with } j_x^r g \in \overline{Y_{y_m}}, j^r g \pitchfork_x^\partial Y\}$$

is an open set of $(C^p(X, X'), T_W(p, t+r))$ and $\{g \in C^p(X, X') \ / \ j^r g \pitchfork^\partial Y\} = \bigcap_{m \in I\!N} T_m^r$.

We have $(I\!R^n)^+_{\Lambda_{y_m}} \supseteq \varphi_{y_m}(U_{y_m}) \supseteq \varphi_{y_m}(s(\overline{Y_{y_m}}))$ and therefore there exist open sets A_m, B_m of $(I\!R^n)^+_{\Lambda_{y_m}}$ such that $\varphi_{y_m}(s(\overline{Y_{y_m}})) \subseteq A_m \subseteq \overline{A_m} \subseteq B_m \subseteq \overline{B_m} \subseteq \varphi_{y_m}(U_{y_m}) \subseteq$

$(I\!R^n)^+_{\Lambda_{ym}}$, (notice that $(I\!R^n)^+_{\Lambda_{ym}}$ is a normal space) and a C^∞-map $\alpha_m : I\!R^n \longrightarrow [0,1]$ such that $\alpha_m(\overline{A_m}) = \{1\}$ and $\alpha_m((I\!R^n)^+_{\Lambda_{ym}} - B_m) = \{0\}$.

Analogously $\varphi'_{y_m}(b(\overline{Y_{y_m}})) \subseteq \varphi'_{y_m}(U'_{y_m}) \subseteq I\!R^{n'_{y_m}}$ and therefore there exist open sets A'_m, B'_m of $I\!R^{n'_{y_m}}$ such that $\varphi'_{y_m}(b(\overline{Y_{y_m}})) \subseteq A'_m \subseteq \overline{A'_m} \subseteq B'_m \subseteq \overline{B'_m} \subseteq \varphi'_{y_m}(U'_{y_m}) \subseteq I\!R^{n'_{y_m}}$ and a C^∞-map $\alpha'_m : I\!R^{n'_{y_m}} \longrightarrow [0,1]$ such that $\alpha'_m(y') = 1$ if $y' \in \overline{A'_m}$, $\alpha'_m(y') = 0$ if $y' \notin B'_m$.

Consider $f \in C^p(X, X')$, $M'_m = I\!R^{n'_{y_m}} \times L(I\!R^n, I\!R^{n'_{y_m}}) \times \ldots \times L^r_s(I\!R^n, I\!R^{n'_{y_m}})$ and an open neighbourhood $\overline{Y}^*_{y_m}$ of $\overline{Y_{y_m}}$ in Y such that $\varphi_{y_m}(s(\overline{Y}^*_{y_m})) \subseteq A_m$ and $\overline{\varphi'_{y_m}(b(\overline{Y}^*_{y_m})} \subseteq A'_m$. Let us consider a compact set C_m of $I\!R^n$ such that $\varphi_{y_m}(U_{y_m}) \subseteq C_m \subseteq I\!R^n$ and the continuous map β_m defined by the following diagram:

$$\beta_m : I\!R^n \times M'_m \xrightarrow{\gamma_m} I\!R^{n'_{y_m}} \xrightarrow{\|\ \|} I\!R^+ \cup \{0\},$$

where $\gamma_m(x, m'_o, m'_1, \ldots, m'_r) = m'_o + m'_1(x) + \ldots + m'_r(x, \overset{(r}{\ldots}, x)$.

Since $\overline{B'_m}$ is a compact set, one has $\varepsilon'_m = D(\overline{B'_m}, I\!R^{n'_{y_m}} - \varphi'_{y_m}(U'_{y_m})) > 0$ and there exists an open neighbourhood M_m of 0 in M'_m such that $\beta_m(y, m') < \varepsilon'_m$ for every $y \in C_m$ and every $m' \in M_m$. Hence if $x \in U_{y_m}$ and $f(x) \in U'_{y_m}$,

$$\lambda_m(m', x) = \alpha_m(\varphi_{y_m}(x)) \cdot \alpha'_m(\varphi'_{y_m}(f(x))) \cdot m'(\varphi_{y_m}(x)) + \varphi'_{y_m}(f(x)) \in \varphi'_{y_m}(U'_{y_m})$$

for every $m' \in M_m$, where $m' = (m'_o, m'_1, \ldots, m'_r)$ and $m'(y) = \sum_{i=0}^{i=r} m'_i(y, \overset{(i}{\ldots}, y)$.

Now we define the map $\varrho_m : M_m \longrightarrow C^p(X, X')$ by:

$$\varrho_m(m')(x) = \begin{cases} f(x) & \text{if} \quad x \notin U_{y_m} \quad \text{or} \quad f(x) \notin U'_{y_m} \\ (\varphi'_{y_m})^{-1}(\lambda_m(m', x)) & \text{if} \quad x \in U_{y_m} \quad \text{and} \quad f(x) \in U'_{y_m} \end{cases}$$

Of course, since $S_m = \varphi^{-1}_{y_m}(supp(\alpha_m)) \cap f^{-1}((\varphi'_{y_m})^{-1}(supp(\alpha'_m))) \subseteq U_{y_m} \cap f^{-1}(U'_{y_m})$, S_m is closed in X and for every $x \in U_{y_m} \cap f^{-1}(U'_{y_m}) - S_m$, $(\varphi'_{y_m})^{-1}(\lambda_m(m', x)) = f(x)$, we have that $\varrho_m(m')$ is a C^p-map. Moreover ϱ_m is a C^p-representation since $e_{\varrho_m} : M_m \times X \longrightarrow X'$ is a C^p-map. Then the map $j^r\varrho_m : M_m \longrightarrow C^{p-r}(X, J^r(X, X'))$ defined by $(j^r\varrho_m)(m') = j^r\varrho_m(m')$, is a C^{p-r}-representation, which follows easily by taking charts.

Let us consider

$$\varepsilon_m = \tfrac{1}{2} min\{D(supp(\alpha'_m), I\!R^{n'_{y_m}} - \varphi'_{y_m}(U'_{y_m})), D(\overline{\varphi'_{y_m}(b(\overline{Y}^*_{y_m}))}, I\!R^{n'_{y_m}} - A'_m)\} \text{ and}$$
$$M^*_m = \{m' \in M_m \,/\, \|m'(\varphi_{y_m}(x))\| < \varepsilon_m \text{ for every } x \in U_{y_m} \text{such that } \varphi_{y_m}(x) \in supp(\alpha_m)\}.$$

Then M^*_m is an open set of M_m and $0 \in M^*_m$. Let us consider $(m', x) \in (M^*_m \times X) \cap (e_{j^r\varrho_m})^{-1}(\overline{Y}^*_{y_m})$. We have the pair $(x, \varrho_m(m')(x))$ belongs to $U_{y_m} \times U'_{y_m}$ and $\varphi_{y_m}(x) \in A_m$.

On the other hand $f(x) \in U'_{y_m}$ and we have

$$\tau_m = d(\varphi'_{y_m}(f(x)), \varphi'_{y_m}(\varrho_m(m')(x))) = d(\varphi'_{y_m}(f(x)), \lambda_m(m', x)) \le \|m'(\varphi_{y_m}(x))\| < \varepsilon_m.$$

Since $\varphi'_{y_m}(\varrho_m(m')(x)) \in \varphi'_{y_m}(b(\overline{Y}^*_{y_m}))$ and $\varepsilon_m < D(\overline{\varphi'_{y_m}(b(\overline{Y}^*_{y_m}))}, I\!R^{n'_{y_m}} - A'_m)$, we have $\varphi'_{y_m}(f(x)) \in A'_m$. Since $\varphi'_{y_m}(f(x)) \in A'_m$ and $\varphi_{y_m}(x) \in A_m$, there exists an open neighbourhood V^x_m of x in $(\varphi_{y_m})^{-1}(A_m) \cap f^{-1}(U'_{y_m})$ such that $\varphi'_{y_m}(f(V^x_m)) \subseteq A'_m$. Thus for every $(m'', y) \in M^*_m \times V^x_m$, we have $e_{j^r\varrho_m}(m'', y) = j^r_y\varrho_m(m'') \in J^r(V^x_m, U'_{y_m})$ and

$$\varphi'_{y_m}(\varrho_m(m'')(y)) = \lambda_m(m'', y) = m''(\varphi_{y_m}(y)) + \varphi'_{y_m}(f(y))$$

Consider the following commutative diagram:

$$\begin{tikzcd}
M^*_m \times V^x_m \arrow[r, "e_{j^r\varrho_m}"] \arrow[d, "1\times\varphi_{y_m}"'] & J^r(V^x_m, U'_{y_m}) \arrow[d, "\pi^{c'_{ym}}_{c_{ym}}"] \\
M^*_m \times \varphi_{y_m}(V^x_m) \arrow[r, "h"'] & \varphi_{y_m}(V^x_m)\times\varphi'_{y_m}(U'_{y_m})\times L(I\!\!R^n, I\!\!R^{n'_{ym}})\times\ldots\times L^r_s(I\!\!R^n, I\!\!R^{n'_{ym}})
\end{tikzcd}$$

where $h(m'', y) = (y, t_{m''}(y), Dt_{m''}(y), \ldots, D^r t_{m''}(y))$, being

$$t_{m''}(y) = m''(y) + \varphi'_{y_m}(f(\varphi^{-1}_{y_m}(y))) = \varphi'_{y_m}(\varrho_m(m'')(\varphi^{-1}_{y_m}(y)))$$

for every $m'' \in M^*_m$ and every $y \in \varphi_{y_m}(V^x_m)$. Then the map defined similarly to h

$$\overline{h} : M'_m \times \varphi_{y_m}(V^x_m) \longrightarrow \varphi_{y_m}(V^x_m) \times I\!\!R^{n'_{ym}} \times L(I\!\!R^n, I\!\!R^{n'_{ym}}) \times \ldots \times L^r_s(I\!\!R^n, I\!\!R^{n'_{ym}})$$

is a C^{p-r}-diffeomorphism. Indeed,

$$D^i t_{m'}(y) = \sum_{k=0}^{r-i} \frac{(i+k)!}{k!} m'_{i+k}(.,\overset{(i}{\ldots},.,y,\overset{(k}{\ldots}\,y) + D^i\lambda(y)$$

for every $i = 1, \ldots, r$, being $\lambda = \varphi'_{y_m} \circ f \circ (\varphi_{y_m})^{-1}$ and $m' = (m'_o, \ldots, m'_r)$. It is clear that $\overline{h}$ is bijective.

Moreover $(\overline{h})^{-1}(\overline{y}, \overline{m_o}, \overline{m}_1, \ldots, \overline{m_r}) = (m'_o, m'_1, \ldots, m'_r, y)$, where $y = \bar{y}$ and for every $i = 0, \ldots, r$

$$m'_i = \frac{1}{i!}\sum_{p=0}^{r-i}(-1)^p \frac{1}{p!}[\overline{m}_{i+p}(.,\overset{(i}{\ldots},.,\bar{y},\overset{(p}{\ldots}\,\bar{y}) - D^{i+p}\lambda(\bar{y})(.,\overset{(i}{\ldots},.,\bar{y},\overset{(p}{\ldots}\,\bar{y})].$$

Thus h is a diffeomorphism from $M^*_m \times \varphi_{y_m}(V^x_m)$ onto its image and therefore $e_{j^r\varrho_m} : M^*_m \times V^x_m \to J^r(V^x_m, U'_{y_m})$ is a C^{p-r}-diffeomorphism onto its image which is an open set. Then $e_{j^r\varrho_m} : M^*_m \times X \to J^r(X, X')$ is infinitesimally transversal to $\overline{Y}^*_{y_m}$ at every $(m', x) \in M^*_m \times X$ such that $e_{j^r\varrho_m}(m', x) = j^r_x(\varrho_m(m')) \in \overline{Y}^*_{y_m}$ and, by the proposition 2.2, $e_{j^r\varrho_m} \pitchfork^\partial \overline{Y}^*_{y_m}$ and $e_{j^r\varrho_m} \pitchfork^* \overline{Y}^*_{y_m}$. Thus, by the theorem 2.1, $K = \{m' \in M^*_m \;/\; j^r\varrho_m(m') \pitchfork^\partial \overline{Y}^*_{y_m}\}$ is residual, hence dense, in M^*_m. Then there exists a sequence $\{m'_p\}_{p\in I\!\!N}$ of elements of K which converges to 0.

Now we have that $\varrho_m(m'_p)_{|X-\overline{U_{y_m}}} = f_{X-\overline{U_{y_m}}}$ for every $p \in I\!\!N$. On the other hand the map $j^{r+t}\varrho_m : M^*_m \longrightarrow C^{p-(r+t)}(X, J^{r+t}(X, X'))$, defined by $(j^{r+t}\varrho_m)(m') = j^{r+t}\varrho_m(m')$, is continuous with the compact-open topology because $e_{j^{r+t}\varrho_m} : M^*_m \times X \longrightarrow J^{r+t}(X, X')$ is continuous . Thus $\{j^{r+t}\varrho_m(m'_p)\}_{p\in I\!\!N}$ converges to $j^{r+t}\varrho_m(0) = j^{r+t}f$ with the compact-open topology and $\{j^{r+t}\varrho_m(m'_p)_{|\overline{U_{y_m}}}\}_{p\in I\!\!N}$ converges uniformly to $(j^{r+t}f)_{\overline{U_{y_m}}}$.

Now $\{\varrho_m(m'_p)\}_{p\in I\!\!N}$ converges to $\varrho_m(0) = f$ in $(C^p(X, X'), T_W(p, r+t))$ *(see [8])*.

Consequently $\{g \in C^p(X, X') \;/\; j^r g \pitchfork^\partial \overline{Y}^*_{y_m}\}$ is a dense set in $(C^p(X, X'), T_W(p, r+t))$. Finally $\{g \in C^p(X, X') \;/\; j^r g \pitchfork^\partial \overline{Y}^*_{y_m}\} \subseteq T^r_m$,and $\{g \in C^p(X, X') \;/\; j^r g \pitchfork^\partial Y\} = \{g \in C^p(X, X') \;/\; j^r g \pitchfork^* Y\} = \bigcap_{m\in I\!\!N} T^r_m$. $\square$

If we have "Y *is a neat* C^{p-r}*-submanifold of* $J^r(X, X')$ *such that, for every* $y \in Y$, $(T_y j)(T_y Y) \subseteq (T_y J^r(X, X'))^i$" then "$Y$ *is a* C^{p-r}*-submanifold of* $J^r(X, X')$ *without boundary*".

For transversality, using the parametrized density theorem [8] and the corollaries 3.1 and 3.4, in the same way that the above theorem, one obtains the following result [8].

Theorem 3.3 *Let X, X' be Hausdorff second countable C^p-manifolds, $p \in \mathbb{N} \cup \{\infty\}$, and r an integer, $0 \leq r < p$. Suppose that $\partial X' = \emptyset$, $dim_x(X) = n \in \mathbb{N}$ for every $x \in X$ and $dim_{x'}(X') < +\infty$ for every $x' \in X'$. Let Y be a totally neat C^{p-r}-submanifold of $J^r(X, X')$ with constant finite codimension q verifying $p - r - 1 \geq n - q$ and $t \in \mathbb{Z}$ with $1 \leq t \leq p - r$. Then $T_Y^r = \{g \in C^p(X, X') \ / \ j^r g \pitchfork Y\}$ is a residual, hence dense, set in $(C^p(X, X'), T_W(p, r+t))$. Moreover, $T_Y^r \subseteq (T^*)_Y^r = \{g \in C^p(X, X') \ / \ j^r g \pitchfork^* Y\}$, and therefore $(T^*)_Y^r$ is a residual, hence dense, set of $(C^p(X, X'), T_W(p, r+t))$.*

The propositions 2.2, 3.2, 3.4 and the theorem 3.2 are also true if we put "*there exist a topological supplement F_y^* of $(T_y j)(T_y Y)$ in $T_y Z$ and a finite linearly independent system Λ_y^* of $L(F_y^*, \mathbb{R})$ such that $(T_y Z)^i \subseteq (T_y j)(T_y Y) + (F_y^*)^+_{\Lambda_y^*}$ and $\partial(T_y Z)^i \subseteq (T_y j)(T_y Y) + \partial(F_y^*)^+_{\Lambda_y^*}$,*" instead of "$(T_y j)(T_y Y) \subseteq (T_y Z)^i$", being $Z = X'$ in the propositions 2.2, 3.2 and $Z = J^r(X, X')$ in the proposition 3.4 and the theorem 3.2 *(see [4])*.

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Received 26 August 1992