ON A CYLINDRICAL BENDING OF A PRISMATIC SHELL WITH TWO CUSPED EDGES UNDER ACTION OF AN IDEAL FLUID

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1. Some General Remarks on Solid-Fluid Transmission Conditions (G.Jaiani).
In the three-dimensional Solid-Fluid models usually the following transmission conditions are used [1]:

$$[u] := u^s - u^f = 0, \quad [\sigma_{ij} n_j] := \sigma^s_{ij} n_j - \sigma^f_{ij} n_j = 0, \tag{1}$$

where u^s, σ^s_{ij} and u^f, σ^f_{ij} are displacement vector and stress tensor in the solid and fluid parts correspondingly, n is outward normal of $\partial(\Omega \backslash I)$, $\Omega = \Omega^s \cup \Omega^f$, I is interface between solid part Ω^s and fluid part Ω^f.

The Solid-Fluid model with unilateral transmission conditions as follows:

$$[u_i N_i] \geq 0, \quad \sigma^f_{ij} n_j = \sigma^f_{NN} N_i, \tag{2}$$

$$\sigma^s_{ij} n_j = -\sigma^f_{NN} N_i, \tag{3}$$

$$\sigma^f_{NN} \leq 0, \tag{4}$$

$$and \ \ \sigma^f_{NN} = 0 \ \ if \ \ [u_i N_i] > 0 \tag{5}$$

has been investigated in [2]. (2) and (3) mean that stresses on interface I are normal (tangential components are equal to zero, σ_{NN} is a normal component, N is an outward normal to Ω^f on I) and balancing each other, (4) means that we have compression, (5) means that if crack between solid and fluid parts is open than at that point normal force equals zero.

Our aim is to construct transmission conditions between three dimensional fluid part, and solid part of plate or shell type, e.g. based on the classical bending theory of plates. Since in this case all mechanical quantities in solid part can be calculated by deflection, as transmission conditions can be used either (1) or (2)-(5). Another possibility of transmission conditions. If middle plane of the plate lies in the plane $0x_1x_2$ and flow of moving fluid is parallel to $0x_3$, it could also have the form:

$$\sigma^f_{33}\left(x_1, x_2, \overset{(+)}{h}(x_1,x_2), t\right) - \sigma^f_{33}\left(x_1, x_2, \overset{(-)}{h}(x_1,x_2), t\right) = q(x_1,x_2,t),$$
$$v_3\Bigg(x_1 - \overset{(+)}{h}(x_1,x_2) w_{,1}(x_1,x_2,t), x_2 - \overset{(+)}{h}(x_1,x_2) w_{,2}(x_1,x_2,t), \overset{(+)}{h}(x_1,x_2) +$$
$$+ w(x_1,x_2,t), t\Bigg) = v_3\Bigg(x_1 - \overset{(-)}{h}(x_1,x_2) w_{,1}(x_1,x_2,t), x_2 - \tag{6}$$
$$- \overset{(-)}{h}(x_1,x_2) w_{,2}(x_1,x_2,t), \overset{(-)}{h}(x_1,x_2) - w(x_1,x_2,t), t\Bigg) = \frac{\partial w(x_1,x_2,t)}{\partial t},$$

(the first of the last pair of equalities is valid since deflection of plate w is independent of x_3), where $(x_1, x_2) \in \Omega$, Ω is projection of plate before deformation, $q(x_1, x_2, t)$ is intensivity of lateral load, $v := (v_1, v_2, v_3)$ is a velocity vector of fluid (compare (6) with the transmission conditions considered in [3], where motion of fluid in elastic pipe has been investigated). On $\partial\Omega$ admissible boundary conditions of plate theory should be given.

2. Title Problem (N. Chinchaladze).

Let the projection on the plane $0x_1x_2$ of plate under consideration be

$$\Omega = \{(x_1; x_2) : -\infty < x_1 < +\infty;\ 0 \le x_2 \le b\}.$$

Let us consider the interface problem of the interaction of the plate, whose flexural rigidity is given by the equation

$$D = D_0 x_2^{\alpha}(b - x_2)^{\beta}, \quad D_0,\ \alpha,\ \beta = const > 0,$$

caused by the thickness

$$2h(x_2) = h_0 x_2^{\alpha/3}(b - x_2)^{\beta/3}, \quad h_0 = const > 0,$$

and of a flow of the incompressible ideal fluid.

Let us consider the case of the flow of the fluid independent of x_1 and parallel to $0x_3$ (directed upwards), with the following conditions at infinity

$$v_3(x_2, x_3, t)|_{x_2^2+x_3^2\to\infty} = v_{3\infty}(t), \quad v_2(x_2, x_3, t)|_{x_2^2+x_3^2\to\infty} = 0,$$

$$p(x_2, x_3, t)|_{x_2^2+x_3^2\to\infty} = p_\infty(t).$$

The transmission conditions in a view of smallness of deflection and of smallness of plate's thickness, taking into account (6), are

$$v_3(x_2, 0, t) = \frac{\partial w(x_2, t)}{\partial t}, \tag{7}$$

$$-p(x_2, \overset{(+)}{h}(x_2), t) + p(x_2, \overset{(-)}{h}(x_2), t) = q(x_2, t). \tag{8}$$

In case of the potential motion of the flow there exists a complex function $\Phi = \psi + i\varphi$, where

$$\frac{\partial\varphi}{\partial x_2} = \frac{\partial\psi}{\partial x_3} = v_2, \quad \frac{\partial\varphi}{\partial x_3} = -\frac{\partial\psi}{\partial x_2} = v_3. \tag{9}$$

Pressure is given by the equation

$$p(x_2, x_3, t) = \rho^f\left[\frac{v_\infty^2}{2} + \frac{p_\infty}{\rho} + \frac{\partial\varphi_\infty}{\partial t} - \frac{\partial\varphi}{\partial t} - \frac{1}{2}(v_2^2 + v_3^2)\right]. \tag{10}$$

In case under consideration for $w(x_2,t)$, in virtue of (8), we get the following differential equation [4]

$$\frac{d^2}{dx_2^2}\left[x_2^{\alpha}(b-x_2)^{\beta}w''_{x_2}(x_2,t)\right]=\frac{2\rho^s h}{D_0}\frac{\partial^2 w(x_2,t)}{\partial t^2}+$$
$$+\frac{-p\left(x_2,\frac{1}{2}h_0x_2^{\alpha/3}(b-x_2)^{\beta/3},t\right)+p\left(x_2,-\frac{1}{2}h_0x_2^{\alpha/3}(b-x_2)^{\beta/3},t\right)}{D_0}. \tag{11}$$

For $\Phi'_{x_2}(x_2,x_3,t)=-v_3+iv_2$, in view of (7), we get the following expression (see[5])

$$\Phi'_{x_2}=-\frac{1}{\pi i\sqrt{(x_2+ix_3)(x_2+ix_3-b)}}\int\limits_{x_0}^{x_2}\frac{\sqrt{(\tau_2+ix_3)(\tau_2+ix_3-b)}}{(\tau_2-x_2)-ix_3}\frac{\partial w(\tau_2,t)}{\partial t}d\tau_2+$$
$$+v_{3\infty}\frac{x_2+ix_3-b/2}{\sqrt{(x_2+ix_3)(x_2+ix_3-b)}}. \tag{12}$$

Let

$$w(x_2,t)=e^{i\omega t}w_0(x_2),\quad p(x_2,x_3,t)=e^{i\omega t}p_0(x_2,x_3),\quad \psi(x_2,x_3,t)=e^{i\omega t}\psi_0(x_2,x_3),$$
$$v_2(x_2,x_3,t)=e^{i\omega t}v_2^0(x_2,x_3),\quad v_3(x_2,x_3,t)=e^{i\omega t}v_3^0(x_2,x_3). \tag{13}$$

If velocity of the flow is too small, then, in virtue of (10), p can be represented in the form (compare [3], p. 14)

$$p(x_2,x_3,t)=\rho^f\left[\frac{p_\infty}{\rho}+\frac{\partial\varphi_\infty}{\partial t}-\frac{\partial\varphi}{\partial t}\right]. \tag{14}$$

From (12), we have expression for v_3. By means of the latter, in view of (9), we can calculate φ which we have to substitute in (14).

Finally, taking into account (14), and (13) from (11) after four times integration, for $w_0(x_2)$ we get the following integral equation

$$w_0(x_2)+\frac{\omega^2}{D_0\pi}\int\limits_{x_2^0}^{x_2}K(\tau_2,x_2)w_0(\tau_2)d\tau_2=f(x_2), \tag{15}$$

where

$$f(x_2)=c_3(x_2-x_2^0)+c_4+\int\limits_{x_2^0}^{x_2}\frac{(c_1+c_2)y(x_2-y)}{y^\alpha(b-y)^\beta}dy-$$
$$-\frac{\omega^2\rho^f}{\pi D_0}\int\limits_{x_2^0}^{x_2}\frac{(x_2-y)dy}{y^\alpha(b-y)^\beta}\int\limits_{x_2^0}^{y}d\tau\int\limits_{-\overset{(+)}{h}(\tau)}^{\overset{(+)}{h}(\tau)}\left\{(\tau-b/2)cos\frac{\phi(\tau,\tau_3)}{2}+\tau_3 sin\frac{\phi(\tau,\tau_3)}{2}\right\}\frac{v_{3\infty}^0 d\tau_3}{\sqrt{r(\tau,\tau_3)}},$$

$$K(\tau_2, x_2) = \int\limits_{\tau_2}^{x_2} \frac{x_2 - y}{y^\alpha (b-y)^\beta} \left\{ -2(y-\tau_2)\pi\rho^s h(\tau_2) + \rho^f \int\limits_{x_2^0}^{\tau_2} (y-\tau)d\tau \int\limits_{-\overset{(+)}{h}(\tau)}^{\overset{(+)}{h}(\tau)} \frac{\sqrt{r(\tau_2,\tau_3)}}{\sqrt{r(\tau,\tau_3)}} \times \right.$$

$$\left. \times \frac{(\tau-\tau_2)cos\{(\phi(\tau_2,\tau_3)-\phi(\tau,\tau_3))/2\} + \tau_3 sin\{(\phi(\tau_2,\tau_3)-\phi(\tau,\tau_3))/2\}}{(\tau_2-\tau)^2+\tau_3^2} d\tau_3 \right\} dy,$$

$$cos\phi(x_2,x_3) = (x_2^2 - x_3^2 - bx_2)/r(x_2,x_3),$$

$$r(x_2,x_3) = \sqrt{(x_2^2 - x_3^2 - bx_2)^2 + (2x_2-b)x_3^2}.$$

The integral equation (15) can be solved by method of successive approximations, and then constants c_i $(i = 1, ..., 4)$ will be defined from the admissible boundary value conditions [4]:

1.1. $w_0(0) = \frac{dw_0(0)}{dy} = w_0(b) = \frac{dw_0(b)}{dy} = 0$, when $0 < \alpha,\ \beta < 1$;

1.2. $w_0(0) = \frac{dw_0(0)}{dy} = \frac{dw_0(b)}{dy} = Q_2(b) = 0$, when $0 < \alpha,\ \beta < 1$;

1.3. $w_0(0) = \frac{dw_0(0)}{dy} = w_0(b) = M_2(b) = 0$, when $0 < \alpha < 1,\ \ 0 < \beta < 2$;

1.4. $w_0(0) = \frac{dw_0(0)}{dy} = Q_2(b) = M_2(b) = 0$, when $0 < \alpha < 1,\ \ \beta > 0$;

2.1. $\frac{dw_0(0)}{dy} = Q_2(0) = w_0(b) = \frac{dw_0(b)}{dy} = 0$, when $0 < \alpha,\ \beta < 1$;

2.2. $\frac{dw_0(0)}{dy} = Q_2(0) = w_0(b) = M_2(b) = 0$, when $0 < \alpha < 1,\ \ 0 < \beta < 2$;

3.1. $w_0(0) = M_2(0) = w_0(b) = \frac{dw_0(b)}{dy} = 0$, when $0 < \alpha < 2,\ \ 0 < \beta < 1$;

3.2. $w_0(0) = M_2(0) = \frac{dw_0(0)}{dy} = Q_2(b) = 0$, when $0 < \alpha < 2,\ \ 0 < \beta < 1$;

3.3. $w_0(0) = M_2(0) = w_0(b) = M_2(b) = 0$ when $0 < \alpha,\ \ \beta < 2$;

4.1. $M_2(0) = Q_2(0) = w_0(b) = \frac{dw_0(b)}{dy} = 0$, when $\alpha > 0,\ \ 0 < \beta < 1$,

where M_2 is the bending moment, Q_2 is the intersecting force.

In the stationary case (i.e. $\frac{\partial w}{\partial t} = 0$) symmetrical prismatic shell $\left(\overset{(+)}{h}(x_2) = = -\overset{(-)}{h}(x_2)\right)$ acts as absolutely rigid $(q(x_2) \equiv 0)$.

R e f e r e n c e s

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