Electronic Journal: Southwest Journal of Pure and Applied Mathematics
Internet: http://rattler.cameron.edu/swjpam.html
ISSN 1083-0464
Vol. No. 01, 1996, pp. 1–15.
Submitted: November 11,1995. Published: April 5, 1996.

Algorithms for Dynamic Negotiation

Aimeé Bella, Ferenc Szidarovszky, and Lucien Duckstein

Abstract

Most conflict situations are dynamic processes where negotiation is carried out and the payoff not only depends on the final state but on the entire process leading to that final state. In this paper, we present three techniques by generalizing the concepts of conflict and solution method so as to include dynamic features into the negotiation methodology. The three techniques are: a graph concept in a dynamic framework, a utility gain model and a negotiation algorithm. In an upper Rio Grande basin case study, we use the negotiation algorithms to resolve conflicts in water resources management, especially water allocation where sources of conflicts appear to involve:

1. satisfaction of water demand,
2. preservation of water quality, and
3. the non-quantification of Native American rights.

Results of the case study are analyzed and interpreted showing how negotiations can lead to satisfactory solutions.

(1991) A.M.S. (MOS) Subject Classification Codes. 90, 93
Key words and Phrases. Conflict, solution concepts, dynamic process, negotiation, graph concept, utility gained model, water allocation, Upper Rio Grande basin.

INTRODUCTION

In this paper, we provide three appproaches for conflict resolution, namely a graph concept in a dynamic framework, a utility gained model, and a negotiation algorithm. We are mainly concerned with conflict resolution and negotiation considered as evolutionary processes involving dynamic changes. Conflict resolution and negotiation are virtually inevitable whenever humans interact,

Systems and Industrial Engineering Department, University of Arizona,
Tucson, AZ 85721, USA
E-mail Address: szidar@sie.arizona.edu

either individually or in groups. A conflict is a situation in which there is a "condition of opposition" and parties with opposing goals affect one another. In other words, in a conflict, two or more decision makers or decision making groups are in dispute over some issue and the resolution depends on the choices of all participants. Conflicts which may be either simple, two-player, complex, or multi-player ones (Bella and Szidarovszky, 1992), can range from a small scale dispute between individuals, to a large military confrontation where huge armies are involved. However, the underlying philosophy and mathematical structure that describe conflict situations have many similar features regardless of the size of the conflicting parties. Fraser and Hipel (1984) have shown that it is often very convenient to model real-world conflicts by describing the possible states, the players' preferences over these states, the inter-state transitions and the transition control by the players. Preferences among the feasible outcomes can be realistically modeled using a minimum amount of ordinal information. As an example, in conflict negotiation, only relative preference information is required when a given player decides that one outcome is preferred over another and it is not necessary to know by how much a player prefers a particular outcome. Therefore, real numbers or utilities do not have to be always assigned to each outcome for representing the preferences of a player and for modeling his/her behavior in a conflict situation. The concepts of conflicts and conflict resolution techniques are also discussed for example by Fang et al. (1993) who give a comprehensive summary of solution concepts based on stability conditions, and introduce a general solution algorithm in graph form. In addition, the authors compared the different solution concepts in order to give a guideline to practitioners for selecting the most appropriate solution method in modeling and solving real world problesm. In a dynamic framework, which is the focus of this paper, there are two main approaches to model and analyze the situation. The first approach is to develop a new model each time period that a model parameter changes (Rapoport, 1969). The second approach taken here incorporates formally the time as a variable in a conflict model by using a graph concept in which at each time period, players make moves by taking into account:

(i) past history of previous moves,

(ii) a utility gain model based on utility theory and

(iii) a negotiation algorithm in which case moves at each time period are done by the weakest player in some sense.

Therefore, the paper is intended to be another contribution in the direction of enriching the application of game theoretic-methods to real-world conflict situations. Specifically, players can make choices (moves) which would interact with those of other players in determining the state of the system in future time periods. We have to point out that to the best of our knowledge, the possibility that the payoff matrix and the relationships between the system states (such as the state transition relations) during the negotiation process could also change over time have not been recognized, or at least no formal methods for coping with such changes have been introduced in the literature. We contend that many water resources conflicts are best regarded as non-stationary processes, in which the system state-payoff matrix relationship is itself subject to change over time. Thus, we generalize the conflict resolution concepts and solution algorithms in a graph theoretical framework, utility gain model and a negotiation process in order to include the above mentioned dynamic features into the methodology.

The paper is organized as follows. In section 1, we present the graph concept in dynamic framework where players' moves are decided according to past history of previous moves. Section 2 develops the utility gain model in which the conflict is carried out by computing a utility gain for each decision maker when he/she decides to change his/her strategy. In section 3, we provide a negotiation scheme showing when and how the decision makers make their moves. Finally, in section 4, using the negotiation algorithm of the previous section, the upper Rio Grande Basin conflict is modeled and solved using water allocation conflict model that involves fifteen (15) decision makers.

1 GRAPH CONCEPT IN DYNAMIC FRAMEWORK

In applying any conflict analysis methodology, the available information on the dispute has first to be organized, and then possible solutions of the dispute have to be constructed. Based on the results, a decision maker can then select actions which would be most beneficial for his/her purposes. The general scheme can be formulated as follows. Let N denote the total number of players, and let $k = 1, 2, \ldots, N$ be the notation of each player. Let $\varphi_k, k = 1, 2, \ldots, N$, denote the payoff function of player k, where

$$\varphi_k : S_1 \times S_2 \times \ldots \times S_N \to R.$$

Here $S_1, \ldots, S_N$ denote the sets of feasible actions of the players. Assume furthermore that the players form G groups, and let $I_j, j = 1, 2, \ldots, N$, denote the set of players in group j. It is also assumed that the cooperation of player k with all other players is expressed as a utility function $\psi_k(\varphi_1, \varphi_2, \ldots, \varphi_N)$, which contains the payoffs of all players. For example, if ψ_k is linear, then it can be written as

$$\psi_k = \varphi_k + \sum_{l \neq k} c_{kl} \varphi_l \tag{1}$$

The payoff of each group j is a function of ϕ_j of the utilities ψ_k of the players belonging to that group. That is, a new game is defined when G is the number of players and the set of strategies of each group is given as Cartesian product of the action sets of the players from that group. In addition, the payoff function of each group is represented as:

$$\phi_j = \phi_j(\psi_{j_i}, \psi_{j_2}, \ldots, \psi_{j_{M(j)}}) \tag{2}$$

where we assume that $\left\{I_j = j_1, j_2, \ldots\ldots, j_{M(j)}\right\}$. Here $M(j)$ denotes the number of players of group j, which might depend on j.

Considering the individual moves of the players, let now the state space be denoted by S, which is usually (but not necessarily) a finite set. The initial state of the conflict is s_o and at each time period $t \geq 0, s_t$ denotes the state at t. Introduce the notation $\mathbf{N} = \{1, 2, \ldots, N\}$. We say that a combination $(i_1, \ldots, i_l; s_1, \ldots, s_l) \in N^l \times S^l$ is feasible, when at $t = 1$, player i_1 can move from s_o to s_1. After this move at $t = 2$, player i_2 can move from s_1 to s_2, etc. Finally, at $t = l$, after all previous moves, player i_l is able to move from S_{l-1} to s_l. Let D_l denote the set of all feasible combinations of length l. At each time period $t \geq 1$, the conflict is given by the combination $(i_1, \ldots, i_t; s_1, \ldots, s_t)$, which shows all moves leading to the final state s_t.

The consequence of any combination $(i_1, \ldots, i_l; s_1, \ldots, s_l)$ to player k is an element of the consequence space Q_k, which is assumed to be partially ordered with binary relation $\Im(k,t)$. For example, "$c\Im(k,t)c^*$" means that player k prefers c^* to c at time period t.

Hence the partial order is time dependent. Introducing the notation

$$D = \bigcup_{l=1}^{\infty} D_l, \tag{3}$$

the payoff function of player k is given as $\beta_k : D \to Q_k$. Notice that for different values of l, β_k has a different number of variables. Therefore β_k depends on time, even if it is not explicitly shown in the notation. Here $\beta_k(i_1, \ldots, i_l; s_1, \ldots, s_l)$ gives the consequence of the entire combination set resulting in the final state s_l to player k. That is, β_k depends on the entire sequence of moves, not only on the final state s_l. We also assume that if $(i_1, \ldots, i_l; s_1, \ldots, s_l) \in D_l$, then for all players $n, (i_1, \ldots, i_l, k; s_1, \ldots, s_l, s_l) \in D_{l+1}$. That is, no change is assumed to be a feasible move to all players. For each time period a directed graph can be defined, where the possible states are the nodes and the possible moves are the arcs. The drawback of this approach is that players need to know the entire feasible combination or the entire history of past moves.

2 UTILITY GAIN MODEL

Let now denote the set of states of $S = \{s_o, s_1, \ldots, S_M\}$, where s_o is the "status quo" and M is the total number of states. Let also $u_i(s_j)$ be the utility of decision maker i if state s_j is selected and $V_{ik}(s_h, s_j)$ be the largest utility value that player k is willing to pay to player i for agreeing in changing state s_h to s_j. Let finally $T_{ik}(s_h, s_j)$ be the maximal threat that k poses on i if he agrees in changing s_h to $s_j, C_{ik}(T)$, the cost of threat T, which is posed on i by k and $W_i(T)$, the utility loss by threat T for i. In the negotation process, decision maker i will agree to change s_h to s_j is he incurs an overall utility gain which has the following components: $\{u_i(s_j) - u_i(s_h)\}$ as the utility gained, $\left\{\sum_{k \neq i} V_{ik} - \sum_{k \neq i} V_{ki}\right\}$ as the received utility minus the lost utility subject to $0 \leq v_{ik} \leq V_{ik}(s_h, s_j)$, $\left\{\sum_{k \neq i} W_{ki}(t_{ik})\right\}$ as total threat received, subject to $0 \leq t_{ik} \leq T_{ik}(s_h, s_j)$ and $\{\sum_{k \neq i} C_{ki}(t_{ki})\}$ as total cost paid for threatening others, subject to $0 \leq t_{ki} \leq T_{ki}(s_h, s_j)$. Hence the total gain is given by:

$$G_i(s_h, s_j) = u_i(s_j) - u_i(s_h) + \sum_{k \neq i} \{v_{ik} - v_{ki} - W_i(t_{ik}) - C_{ki}(t_{ki})\} \tag{4}$$

Therefore, i will agree on changing s_h to s_j if and only if there are feasible values of v_{ik}, v_{ki}, t_{ik}, and t_{ki} such that $G_i(s_h, s_j) \geq 0$.

Assume finally that the current state is s_h. Compute all utility gains $u_i(s_j) - u_i(s_h)$, for player i which are denoted as $u_i(s_j, s_h)$. Let $u_{i^*}(s_{j^*}, s_h)$ be equal to $\max\{u_i(s_j, s_h)\}$. Then player i^* tries to make the move to s_{j^*}. First, he checks on the possibility of convincing all other parties or sufficiently many other parties to agree with this change. Then he suggests a move and tries to convince as many participants as possible by negotiating with them on possible side payments and threats. If successful, then s_{j^*} will become the new state. Otherwise, this move is dropped, the second largest of the numbers $u_i(s_j, s_h)$ is selected, and a new move is examined. This process terminates when no more move is possible. An alternative approach is based on maximizing the relative utility gain

$$\frac{u_i(s_j) - u_i(s_h)}{|u_i(s_h)|}, \tag{5}$$

when relative gain is more important to the players than simple utility increase.

3 NEGOTIATION ALGORITHM

This section describes a negotiation algorithm which can especially be used in resource allocation conflicts. It determines which decision makers should make the concession at each time period. To start, let N denote again the number of players in a conflict. Let x_k denote the demand of player k, and let $u_i(x)$ denote the utility function of player k if simultaneous demand vector $x = (x_1, \ldots, x_N)$ is selected. Assume that not enough resources are available to satisfy all demands, then at least one player will get less than the demand according to the regulations and agreement among players. Let $x^* = (x_1^*, \ldots, x_N^*)$ denote the "status quo" or the amount each player gets if no agreement is achieved. The negotiation procedure is modeled in the following step. At time period $t = 0$, each player selects a demand. If the corresponding simultaneous strategy vector is feasible, then agreement is reached. Otherwise,at least one player changes his/her demand. If the new simultaneous strategy vector becomes feasible, then the negotiation procedure terminates. Otherwise, a new player makes the next move. This procedure continues until a feasible solution is reached. If no agreement is reached, that is, no further moves are made, then the players receive the corresponding components of the disagreement vector. Assume now that at a certain time period of the negotiating process, the current demand vector $x = (x_1, \ldots, x_N)$ is not feasible. Then the players must continue the bargaining process, otherwise, the "status quo" point will be automatically selected, which is unfavorable to all decision makers. The question that arises now is the following: who makes the next move and which offer is made by this player? It is natural to assume that the player who will have the largest relative loss in switching to the "status quo" point should make the next concession, and he has to continue relaxing his position until the new state becomes feasible or someone else becomes the weakest player. After his offer is made, a new negotiation state with a new demand vector will be in effect, and the weakest player at this new stage makes the next move. This procedure continues until a feasible state is found. This concept can be formulated mathematically in the following way. Define for all x, i, and $t \in [0,1]$,

$$Q_i(x,t) = \frac{U_i(x_1,\ldots,x_{i-1},x_i - t(x_i - x_i^*),x_{i+1},\ldots,x_N) - U_i(x^*)}{U_i(x_1,\ldots,x_{i-1},x_i - t(x_i - x_i^*),x_{i+1},\ldots,x_N)} \tag{6}$$

which gives the relative loss of player i by switching the state vector to the "status quo" point. We say that at state x, player i is the weakest, if for some small $\epsilon > 0$,

$$Q_i(x,t) > Q_j(x,t) \tag{7}$$

for all $j \neq i$ and $0 < t \leq \epsilon$.

In this case, player i has to relax his demand from state x_i to $x_i - t(x_i - x_i^*)$, where t is the smallest positive number such that player i is not the weakest anymore. It at a certain state x, there are two or more weakest players, then all of them should relax their demands in this way simultaneously. If the resulting new state is feasible, then the negotiation ends. If the new state is nondominated, then this is considered to be the outcome of the negotiation process. If is is not, then the final outcome is defined as follows. Let x be the state before the last stage. Then define:

$$t^* = \min\left\{t | x + t(x - x^*) \text{ is nondominated}\right\},$$

and select the final outcome as $x + t^*(x - x^*)$. Notice that an "equity" type adjustment is made is this final step.

4 NUMERICAL APPLICATION

The upper Rio Grande Basin conflict (Figure 1) which involves multiple objectives – multiple decision makers, is modeled using the negotiation algorithm of the previous section. The conflict is over water demand satisfaction scenarios. In other words, we model how available water supplies should be shared among all users. The conflict involves 15 major decision makers (Table 1) and 30 plausible option strategies (Bella et al., 1995), reduced to 15 major strategies (Table 1) that are hypothetically defined by the authors based on available reports (US Army Corps, 1989 and US Bureau of Reclamation, 1992). Historically, the division of surface waters has been carried out amiably by mutual agreement between the three groups of users: the states of Colorado, New Mexico, and Texas. Specifically, the water balance along the river is monitored and allocated by the US Bureau of Reclamation, the US Army Corps of Engineers, and the International Boundary and Water Commission (IBWC) according to the Rio Grande compact of 1938 and the two US-Mexico agreements of 1906 and 1944. Water consumption in the Rio Grande Basin is primarily for urban, agricultural, and recreational uses, with some water used for hydroelectric power generation and cooling. The largest water-use category is irrigation. The bulk of the irrigation water comes from the river. A significant fraction of the total water use is pumped ground water. Most cities along the valley depend completely on groundwater for domestic use.

The lack of surface water in the past and present management schemes have been made up through interbasin diversions and ground water mining, but extensive ground water withdrawal from stream-aquifer systems has resulted in a reduction of available surface water supplies of the Rio Grande. The current management of water resources is complicated by a multiplicity of governmental units on both sides of the border. On the Mexican side, the Federal government exercises the dominant role through a variety of agencies. On the US side, primary responsibility is placed in three different states, each having its separate, independent and different water laws. Economic activities and population for the region are expected to increase, especially with the recent signing of the North American Free Trade Agreement (NAFTA) with the Republic of Mexico and Canada. These developments, which will place greater demands on water resources, may require changes in public policies presumably through negotiation. Thus, water management in the basin is complex because many of the water needs are incompatible. Allocation changes designed to meet a diminishing water supply would be difficult to implement because of the complex web of international treaties, interstate compacts and local, state and federal laws (Waterstone, 1994). The high degree of complexity makes it difficult for a decision maker to comprehend the problem fully and eventually reach a reasonable decision.

Furthermore, climatologists predict that the increase in greenhouse gases may result in a warmer and drier climate in many parts of the world in the next century. The Upper Rio Grande is one such area and it will be extremely vulnerable to any change that would decrease available supply. Revell and Waggoner (1983) predicted that a 2-degree temperature increase and a 10 percent decrease in annual precipitation would reduce runoff in the Rio Grande river basin by 76 percent. This represents the largest percentage reduction for any basin in the United States (Thomson and Stephen, 1990), and this is the main motivation for having selected this region as a case study.

Sources of water allocation conflicts appear to be as follows (Bella et al., 1995): (1) The absence of agreements on ground water usage and water supply, especially since increased demands and possibly climate change may lead to increased pumpage and a possible lowering of the water table; (2) Water quality problems; (3) The non-quantification of Indian rights, which as a necessary requirement will require an adjudication. The set of selected alternatives or strategy options include the following (Duckstein et al., 1995):

1. (1) increasing water supply and irrigation efficiency,
2. (2) meeting water quality and protecting the public interest,
3. (3) constructing hydropower and enhancing flood control,
4. (4) reducing sediment by pool stabilization, and
5. (5) establishing groundwater regulation.

Combining these actions, we have indentified the set of strategies given in Table 2. If we let Table 3 represent the players' utilities at a certain time period in the negotiation process, then we can compute values of $Q_i(x,t)$ using equation (6). The results are given in Table 4. For example, the values of $Q_1(A_1, x_1)$ and $U_1(A_3, x_1)$ are computed as follows:

$$Q_1(A_1, x_1) = \frac{U_1(A_1, x_1) - U_1(x^*)}{U_1(A_1, x_1)} = \frac{0.5 - 0.1}{0.5} = 0.8$$

$$Q_1(A_3, x_1) = \frac{U_1(A_3, x_1) - U_1(x^*)}{U_1(A_3, x_1)} = \frac{0.2 - 0.1}{0.2} = 0.5$$

Based on these values, the weakest players are identified and underlined in Table 4. For example, since $Q_1(A_4, x_1)$ corresponds to the larger relative loss for strategy x_1, A_4 (or player 4) has to make the concession by relaxing his water demand. For strategy x_5, or "Reducing sediment by pool stabilization", player A_7 or "Bureau of Reclamation" makes the move because he appears to be the weakest player. For strategy x_2, three players, namely player A_4 (Irrigation Districts), player A_{10} (New Mexico Compact Commissioners), and A_{14} (Middle Rio Grande Conservation District) make the move. Zero values in Table 4 indicate the status quo for the player.

5 CONCLUSIONS

In this paper, we have focused on dynamic negotiation algorithms for a conflict situation by presenting three conflict resolution techniques, namely: the graph concept, the utility gain model and a negotiation algorithm. Then, using a case study, the upper Rio Grande Basin conflict, a real world application is given as an example to show how players make moves during the negotiation process. This methodology is highly recommended in practical cases when the relevant data are available.

References

[1] Bella A. A. and Szidarovszky F. Water Allocation Conflict under Climatic Uncertainty: Case of Upper Rio Grande Basin, USA. Pure Math. and Appl. Ser. C., Vol. 3, No. 1-4, pp. 35-58 (1992).

[2] Bella A. A., Duckstein L. and Szidarovszky, F. A Multicriterion Analysis of Water Resources in the Upper Rio Grande Basin. Invited paper, Western Regional Science Association Meeting, San Diego, Feb. 1995.

[3] Brams S. J. Theory of Moves. Cambridge University Press, Cambridge, U.K., 1993

[4] Duckstein L., Bella A., Stakhiv E. Z. and Waterstone M. Conflict Analysis Using Multiple Criterion Decision Making Under Uncertainty In: Proceedings NATO ASI Transboundary Water Resources: Management and Environmental Issues Conference, Skopelos, Greece, May 15-25 (to appear 1995).

[5] Fang L., Hipel K. W. , and Kilgour D. M. Interactive Decision Making: The Graph Model for Conflict Resolution. John Wiley & Sons, Inc., New York, 1993.

[6] Fraser N. M. and Hipel K. W. Conflict Analysis: Models and Resolutions. North-Holland, New York, 1984

[7] Rasmusen E. Games and Information: An Introduction to Game Theory. Basil Blackwell, Oxford, 1989

[8] Rapoport, A. Effects of Payoff Information in Multi-Stage Mixed Motive Games. Behavioral Sci., Vol. 14, 1969, pp. 204-215.

[9] Revell R. R. and Waggoner P. E. Efforts of the Carbon Dioxide Induced Climatic Change on Water Supplies in the Western United States in Changing Climate. Report of the Carbon Dioxide Assessment Committee. National Academy Press, Washington, D.C. 1983.

[10] U.S. Army Corps of Engineers Reevaluation of the Operating Plan. Report: Albuquerque Project Office. Albuquerque, New Mexico. 1989

[11] U.S. Bureau of Reclamation Annual Operating Plan. Report: Albuquerque Projects Office. Albuquerque, New Mexico. 1992.

[12] Thomson B. and Stephen A. Rio Grande Laboratory/ Consortium Concepts. Proceedings of the Rio Grande Basin - Global Climate Change Scenarios, WRRI Rep. M24, 1990, pp. 33-36.

[13] Waterstone M. Institutional Analysis of Water Resources. In: Engineering Risk in Natural Resources Management with Special References to Hydrosystems under changes of Physical or Climatic Environment. L. Duckstein and Parent E. (eds). Kluwer, Amsterdam. Series E: Applied Sciences Vol. 275, 1994 pp. 59-75.

Table 1. Identified decision makers in the Upper Rio Grande

Notation	Decision Makers
A_1	US Forest Service and Land Management
A_2	Pueblo Indian Tribes
A_3	Recreation Groups
A_4	Irrigation Districts
A_5	User Groups
A_6	Army Corps of Engineers
A_7	Bureau of Reclamation
A_8	International Boundary and Water Commission
A_9	Colorado Compact Commissioners
A_{10}	New Mexico Compact Commissioners
A_{11}	Texas Compact Commissioners
A_{12}	Fish and Wildlife Service
A_{13}	Department of Interior
A_{14}	Middle Rio Grande Conservation District
A_{15}	Mexican Government

Table 2. Selected strategies in the Upper Rio Grande Notation & Strategies

Notation	Strategies
X_1	Do nothing
X_2	Increasing water supply and irrigation efficiency
X_3	Meeting water quality and protecting the public interest
X_4	Constructing hydropower and enhancing flood control
X_5	Reducing sediment by pool stabilization
X_6	Establishing groundwater regulation
X_7	Increasing water supply and irrigation efficiency, meeting water quality and protecting the public interest
X_8	Increasing water supply and irrigation efficiency, constructing hydropower and enhancing flood control
X_9	Increasing water supply, irrigation efficiency and reducing sediment by pool stabilization
X_{10}	Increasing water supply and irrigation efficiency and establishing groundwater regulation
X_{11}	Meeting water quality, protecting the public interest, constructing hydropower and enhancing flood control
X_{12}	Meeting water quality, protecting the public interest and reducing sediment by pool stabilization
X_{13}	Meeting water quality, protecting the public interest establishing groundwater regulation
X_{14}	Constructing hydropower, enhancing flood control and reducing sediment by pool stabilization
X_{15}	Constructing hydropower, enhancing flood control and establishing groundwater regulation

Table 3. Utility function of strategies and player

$U_i(x)$	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}	A_{12}	A_{13}	A_{14}	A_{15}
X_1	0.5	0.1	0.2	0.7	0.8	0.5	0.5	0.3	0.6	0.1	0.12	0.1	0.7	0.1	0.1
X_2	0.2	0.6	0.2	0.95	0.9	0.7	0.75	0.5	0.8	0.9	0.75	0.4	0.3	0.9	0.5
X_3	0.85	0.65	0.9	0.4	0.77	0.5	0.55	0.35	0.67	0.6	0.55	0.95	0.33	0.43	0.5
X_4	0.3	0.2	0.4	0.25	0.45	0.9	0.95	0.7	0.6	0.75	0.55	0.4	0.6	0.5	0.3
X_5	0.2	0.4	0.6	0.5	0.65	0.8	0.75	0.3	0.4	0.5	0.45	0.3	0.55	0.7	0.6
X_6	0.1	0.15	0.25	0.3	0.32	0.5	0.45	0.6	0.9	0.85	0.97	0.23	0.7	0.82	0.5
X_7	0.2	0.65	0.6	0.95	0.6	0.5	0.7	0.43	0.8	0.6	0.82	0.9	0.7	0.3	0.5
X_8	0.6	0.5	0.53	0.27	0.4	0.74	0.8	0.3	0.5	0.5	0.6	0.4	0.3	0.5	0.1
X_9	0.1	0.3	0.4	0.7	0.21	0.4	0.35	0.5	0.6	0.7	0.65	0.41	0.5	0.6	0.12
X_{10}	0.83	0.9	0.7	0.1	0.15	0.2	0.3	0.9	0.1	0.3	0.2	0.7	0.12	0.3	0.8
X_{11}	0.2	0.5	0.5	0.8	0.43	0.9	0.1	0.21	0.7	0.6	0.3	0.44	0.6	0.22	0.35
X_{12}	0.11	0.82	0.6	0.22	0.92	0.5	0.5	0.42	0.6	0.61	0.53	0.74	0.3	0.47	0.4
X_{13}	0.12	0.6	0.52	0.46	0.6	0.7	0.4	0.31	0.7	0.6	0.44	0.53	0.4	0.36	0.39
X_{14}	0.3	0.3	0.1	0.5	0.4	0.97	0.91	0.12	0.22	0.4	0.61	0.33	0.94	0.8	0.94
X_{15}	0.45	0.21	0.2	0.3	0.57	0.92	0.87	0.3	0.7	0.5	0.7	0.5	0.5	0.63	0.81

Table 4. Relative loss function of strategies and players

$Q_i(A,X)$	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}	A_{12}	A_{13}	A_{14}	A_{15}
X_1	.8	0	.5	$\underline{.86}$	.81	.6	.8	.6	.83	0	0	0	.83	0	0
X_2	.5	.83	.5	$\underline{.89}$	.83	.71	.87	.76	.87	$\underline{.89}$	.87	.75	.6	$\underline{.89}$	.8
X_3	.88	.85	$\underline{.89}$	.75	.81	.6	.82	.66	.85	.83	.82	$\underline{.89}$	.64	.77	.8
X_4	.67	.5	.75	.6	.67	.78	$\underline{.89}$	.83	.83	.87	.82	.75	.8	.8	.67
X_5	.5	.75	.83	.8	.77	.75	$\underline{.87}$	.6	.75	.8	.78	.67	.78	.86	.83
X_6	0	.33	.6	.67	.53	.6	.78	.8	.89	.88	$\underline{.9}$	.57	.83	.88	.8
X_7	.5	.85	.83	$\underline{.89}$	.78	.6	.86	.72	.87	.83	.88	$\underline{.89}$	.83	.67	.8
X_8	.83	.8	.81	.63	.62	.73	$\underline{.87}$	.6	.8	.8	.83	.75	.6	.8	0
X_9	0	.67	.75	$\underline{.86}$	.29	.5	.75	.76	.83	$\underline{.86}$	.85	.76	.76	.83	.17
X_{10}	.88	$\underline{.89}$	.86	0	0	0	.67	.87	0	.67	.5	.86	0	.67	.88
X_{11}	.5	.8	.8	$\underline{.87}$	.65	.78	0	.43	.86	.83	.67	.77	.8	.55	.71
X_{12}	.09	$\underline{.88}$	.83	.55	.84	.6	.8	.71	.83	.84	.81	.86	.6	.79	.75
X_{13}	.17	.83	.81	.78	.75	.71	.75	.61	$\underline{.86}$	.83	.77	.81	.7	.72	.74
X_{14}	.67	.67	0	.8	.62	.79	$\underline{.89}$	0	.55	.75	.84	.7	.87	.88	$\underline{.89}$
X_{15}	.78	.52	.5	.67	.74	.78	$\underline{.89}$	.6	.86	.8	.86	.8	.76	.84	.88