NORMED ALGEBRAS AND THE GEOMETRIC SERIES TEST

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Abstract. The purpose of this article is to survey a class of normed algebras that share many central features of Banach algebras, save for completeness. The likeness of these algebras to Banach algebras derives from the fact that the geometric series test is valid, whereas the lack of completeness points to the failure of the absolute convergence test for series in the algebra. Our main result is a compendium of conditions that are all equivalent to the validity of the geometric series test for commutative unital normed algebras. Several examples in the final section showcase some incomplete normed algebras for which the geometric series test is valid, and still others for which it is not.

Full text

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