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On the Effective Action of Dressed Mean Fields for $\mathcal{N} = 4$ Super-Yang–Mills Theory

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Abstract. On the basis of the general considerations such as R-operation and Slavnov–Taylor identity we show that the effective action, being understood as Legendre transform of the logarithm of the path integral, possesses particular structure in $\mathcal{N} = 4$ supersymmetric Yang–Mills theory for kernels of the effective action expressed in terms of the dressed effective fields. These dressed effective fields have been introduced in our previous papers as actual variables of the effective action. The concept of dressed effective fields naturally appears in the framework of solution to Slavnov–Taylor identity. The particularity of the structure is independence of these kernels on the ultraviolet regularization scale Λ. These kernels are functions of mutual spacetime distances and of the gauge coupling. The fact that β function in this theory vanishes is used significantly.

Key words: R-operation; gauge symmetry; $\mathcal{N} = 4$ supersymmetry; Slavnov–Taylor identity

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Slavnov–Taylor (ST) identity [1] is an important tool in quantum field theory. It is a consequence of BRST symmetry [2] of the tree level action of gauge theories, and it consists in an equation written for a functional that is called effective action [3]. An approach to solving the ST identity in gauge theories has been proposed recently [4, 5]. In this Letter we re-consider our analysis for a particular case of $\mathcal{N} = 4$ supersymmetric theory. Our analysis will be based on five theoretical tools: R-operation [6], gauge symmetry, $\mathcal{N} = 4$ supersymmetry, the ST identity itself, and absorbing two point Green's functions into a re-definition of the effective fields. Effective fields are variables of the effective action [3].

$\mathcal{N} = 4$ super-Yang–Mills theory is widely considered from the point of view AdS/CFT correspondence [7]. Anomalous dimensions of gauge invariant operators are related to energies of string states [8]. In this Letter we consider $\mathcal{N} = 4$ super-Yang–Mills theory from a different point. We analyze one particle irreducible (proper) correlators of this theory, which are kernels of the effective action. For example, a kernel can be proper vertex of several gluons. We hope this analysis can have application to calculation of maximal helicity violating amplitudes of processes with n gluons [9, 10]. $\mathcal{N} = 4$ super-Yang–Mills theory is useful theoretical playground to understand better the problems that stand in QCD. This model has special particle contents. In addition to one gluon and four Majorana fermions it contains six scalar fields. All the particles are in the adjoint representation of $SU(N)$ gauge group.

It has been known for a long time that for $\mathcal{N} = 4$ Yang–Mills supersymmetric theory beta function of gauge coupling vanishes [11, 12, 13, 14]. We extensively use this fact in our analysis.

The basic notation of this paper coincides with notation of [4]. The main ST identity is [3]

$$\begin{aligned}&\mathrm{Tr}\left[\int dx\,\frac{\delta\Gamma}{\delta A_m(x)}\frac{\delta\Gamma}{\delta K_m(x)}+\int dx\,\frac{\delta\Gamma}{\delta c(x)}\frac{\delta\Gamma}{\delta L(x)}-\int dx\,\frac{\delta\Gamma}{\delta b(x)}\left(\frac{1}{\alpha}\partial_m A_m(x)\right)\right]\\&\quad+\int dx\,\frac{\delta\Gamma}{\delta\phi(x)}\frac{\delta\Gamma}{\delta k(x)}+\int dx\,\frac{\delta\Gamma}{\delta\bar{k}(x)}\frac{\delta\Gamma}{\delta\bar{\phi}(x)}=0.\end{aligned} \tag{1}$$

The effective action is a functional of all the effective fields and external sources participating in this equation, $\Gamma\equiv\Gamma[A_m,b,c,\phi,\bar{\phi},K_m,L,k,\bar{k}]$ [24, 3]. The external sources K_m, L, k, $\bar{k}$ are coupled in the exponential of the path integral to the BRST transformations of fields from the measure of the path integral [3], that is, to the BRST transformations of fields A_m, c, ϕ, $\bar{\phi}$ of the tree level action, respectively. The effective fields A_m, b, c, ϕ, $\bar{\phi}$ are traditionally designated by the same letters as are the fields A_m, b, c, ϕ, $\bar{\phi}$, which are variables of the path integral. The effective fields are defined as variational derivatives of the logarithm of the path integral with respect to the corresponding external sources coupled to these variables of integration in the path integral [3]. The matter effective field ϕ stands for spinors as well as for scalars. We assume summations over all indices of the representation of matter fields. The traditional Lorentz gauge fixing is taken and the corresponding Faddeev–Popov ghost action introduced according to the line of Ref. [13]. These terms break supersymmetry of the tree level action. The β function is zero but anomalous dimensions of propagators are non-zero [13].

Consider the vertex Lcc. Here we do not specify arguments of the effective fields. It is the only vertex, which is invariant with respect to the ST identity at the classical level. At the quantum level it transforms to the form

$$\langle Lcc\rangle\times\langle Lcc\rangle+\langle LccA\rangle\times\langle K_m\partial_m c\rangle=0. \tag{2}$$

This is a direct consequence of the main ST identity (1) and is a schematic form of the ST identity relating the Lcc and $LccA$ field monomials. The precise form of this relation can be obtained by differentiating the identity (1) with respect to L and three times with respect to c and then by setting all the variables of the effective action equal to zero. The brackets in (2) mean that we have taken functional derivatives with respect to fields in the corresponding brackets at different arguments and then have put all the effective fields equal to zero.

We know from the theory of R-operation [6] that in Yang–Mills theory the divergences can be removed by re-defining the fields and the gauge coupling. Thus, there are four renormalization constants that multiply the ghost, gluon, spinor, and scalar fields [3]. The gauge coupling also must be renormalized but this is not the case in the theory under consideration. In this paper we concentrate on two regularizations: regularization by higher derivatives described in [3], Λ is the regularization scale, and regularization by dimensional reduction [15]. The regularization by higher derivatives has been constructed for supersymmetric theories in [16, 17]. Having used this regularization, new scheme has been proposed in [18, 19, 20]. We assume here that the component analog of that scheme can be constructed. The regularization by higher derivatives provides strong suppression of ultraviolet divergences by introducing additional terms with higher degrees of covariant derivatives acting on Yang–Mills tensor into the classical action, which are suppressed by appropriate degrees of the regularization scale Λ. In addition to this, it is necessary to introduce a modification of the Pauli–Villars regularization to guarantee the convergence of the one-loop diagrams [3]. This scheme does not break gauge invariance beyond one loop level. Moreover, it has been suggested in [3] that such modification by Pauli–Villars terms to remove one-loop infinities is gauge invariant by construction. To regularize the fermion cycles, the usual Pauli–Villars regularization can be used. However, this approach, when applied to explicit examples, is known to yield incorrect results in Landau gauge [21]. A number of suggestions have been put forward to treat this problem [22, 23]. As was shown in [23], the contradiction, noticed in [21] is related to the singular character of Landau gauge. In all other covariant gauges the method works, and to include also the Landau gauge one has to add one more Pauli–Villars field to get the correct result [22, 23]. We analyze the theory in the regularization by the dimensional reduction in a parallel way [15].

At one-loop level the part associated with the divergence of Lcc term must be invariant itself under the ST identity since the second term in identity (2) is finite in the limit of removing regularization $\Lambda \to \infty$ [4]. According to Ref. [4], this results in the following integral equation for the part of the correlator Lcc corresponding to the superficial divergence $\sim \ln \frac{p^2}{\Lambda^2}$:

$$\int dx\, \Gamma_\Lambda(y', x, z')\Gamma_\Lambda(x, y, z) = \int dx\, \Gamma_\Lambda(y', y, x)\Gamma_\Lambda(x, z, z')$$
$$= \int dx\, \Gamma_\Lambda(y', x, z)\Gamma_\Lambda(x, z', y), \tag{3}$$

where $\Gamma_\Lambda(x, y, z)$ is this scale($\Lambda-$) dependent part of the most general parametrization $\Gamma(x, y, z)$ of the correlator Lcc,

$$\Gamma \sim \int dx\, dy\, dz\, \Gamma(x, y, z) f^{abc} L^a(x) c^b(y) c^c(z). \tag{4}$$

Here f^{abc} is the group structure constant. The only solution to the integral equation (3) is [4]

$$\Gamma_\Lambda(x, y, z) = \int dx'\, G_c(x' - x)\; G_c^{-1}(x' - y) G_c^{-1}(x' - z). \tag{5}$$

The subscript Λ means scale-dependent part of the correlator. As can be seen, all scale-dependence of this correlator is concentrated in the dressing function. The complete correlator Lcc at one loop level can be then written as

$$\int dx\, dy\, dz\, \Gamma(x, y, z) \frac{i}{2} f^{bca} L^a(x) c^b(y) c^c(z)$$
$$= \int dx'\, dy'\, dz'\, dx\, dy\, dz\, \tilde{\Gamma}(x', y', z') G_c(x' - x) G_c^{-1}(y' - y)$$
$$\times\, G_c^{-1}(z' - z) \frac{i}{2} f^{bca} L^a(x) c^b(y) c^c(z).$$

Here $\tilde{\Gamma}(x', y', z')$ is scale-independent kernel of Lcc correlator[1].

We absorb this dressing function G_c into the corresponding re-definition of the fields L and c, and then divide the ghost propagator into two parts, one of which is related to the dressing function of the ghost field G_c, and we call another the dressing function of the gluon field G_A. The effective field K and the antighost field b get opposite re-definition by integrating with the dressing function G_A^{-1} [4]. The important point here is covariance of the part of the ST identity without gauge fixing term with respect to such redefinitions [4]. In terms of the dressed effective fields, we have a useful relation which is a consequence of the main ST identity (1) and can be obtained by differentiating the main ST identity two times with respect to $\tilde{c}$ and one time with respect to $\tilde{b}$. This resulting identity is

$$\langle \tilde{A}_m \tilde{b} \tilde{c} \rangle \times \langle \tilde{K}_m \tilde{c} \rangle + \langle \tilde{L} \tilde{c} \tilde{c} \rangle \times \langle \tilde{b} \tilde{c} \rangle = 0. \tag{6}$$

Again, this identity is written in a schematic way. However, a new important point appears here. Namely, since the two-point proper functions in terms of dressed effective fields are trivial

[1] In [4] we conjectured, on the basis of equations (3) and (4), that the complete $\Gamma(x, y, z)$ has the structure (5); this would correspond to $\tilde{\Gamma}(x', y', z') \propto \delta(x' - y')\delta(x' - z')$.

tree level two-point proper functions, the divergences of $\langle \tilde{A}_m \tilde{b}\tilde{c}\rangle$ and $\langle \tilde{L}\tilde{c}\tilde{c}\rangle$ coincide. Since $\langle \tilde{L}\tilde{c}\tilde{c}\rangle$ is scale-(or Λ-) independent, the $\langle \tilde{A}_m \tilde{b}\tilde{c}\rangle$ is scale-independent also. Concerning the gluon propagator, one part of the divergence is in the dressing function G_A, and the rest of divergence would be absorbed in the re-definition of the gauge coupling constant. This last divergence is absent in $\mathcal{N}=4$ theories. All the other correlators are solved by the ST identity in terms of the dressed effective fields and their kernels are finite (do not possess divergence in the limit of removing regularization) and scale-independent.

Infinite parts of the dressing functions will be one loop counterterms corresponding to the re-definition of the fields. Then we can repeat this procedure at two loop level and so on, up to any order in loop number. Indeed, re-definition by multiplication of the fields of the tree level action results in re-defining external legs of proper correlators in comparison with the unrenormalized theory. This property was used by Bogoliubov and Shirkov [6] in the derivation of renormalization group equations. Re-defining the effective fields by dressing functions does not bring new aspects in this sense. Indeed, re-defining variables of the path integral by dressing will result in the dressing of the external legs of the proper correlators. By proper correlators we mean kernels of Γ, that is, these kernels are one particle irreducible diagrams. We have already used this re-definition in the dressing functions in the $\tilde{L}\tilde{c}\tilde{c}$ correlator. Thus, new superficial divergences will have to satisfy the integral equation (3) at two loop order too in terms of effective fields $\tilde{L}\tilde{c}\tilde{c}$, which are effective fields dressed by one loop dressing function G_c. We then repeat this re-definition in the same procedure in each order of the perturbation theory as we did in the previous paragraph at one loop level.

Until now the pure gauge sector has been considered. Fermions are necessary for providing supersymmetry. Consider vertex $kc\phi$ at one loop level. The superficial divergence of this vertex is cancelled by the divergence of the vertex Lcc. This means that in the part of the correlator $kc\phi$ corresponding to superficial divergence $\sim \ln \frac{p^2}{\Lambda^2}$ this divergence can be absorbed into the dressing functions in the following way:

$$\int dx\, dx_1\, dy_1\, dy_2\, G_\phi(x-x_1)\ G_c^{-1}(x-y_1) G_\phi^{-1}(x-y_2) k(x_1) c(y_1) \phi(y_2). \tag{7}$$

Note that the dressing functions for the fermions and scalars $G_\phi^{-1}(x)$ are not fixed yet. We set them equal to halves of the two point matter functions. The remaining vertices are restored in the unique way because the ST identity works.

This theory has intrinsic on-shell infrared divergences, like those canceled by bremsstrahlung of soft gluons (such a cancellation happens on shell). To regularize these divergences we can introduce mass parameter μ [25]. Such a trick breaks the ST identity by terms dependent on μ. At the end the dependence on μ will disappear in physical matrix elements. We mean by the physical matrix element a connected diagram on-shell contribution to amplitudes of particles. In comparision, the dimensional reduction regulates both the ultraviolet and the infrared divegences. On the infrared side, this is better than to introduce small mass parameter μ, the reason being that in a non-Abelian gauge theory the limit for infrared regulator going to zero is in general singular. Infrared divergences cancel in the physical matrix elements in dimensional reduction too. However, one can think that in the effective action (off shell) they could be present since cancellation of the infrared divergences happens (on shell) between proper and one particle reducible graphs [25]. Below we will indicate that off shell these infrared divergences do not exist at all in the position space[2].

In principle, infrared divergences in the effective action represent an outstanding problem that is not treated in the present work in necessary details. We show in this paragraph that in the regularization by the dimensional reduction this problem does not appear. It is enough

[2]We treat the theory in the position space. Infrared divergences are absent off shell also in the momentum space by the same reasons.

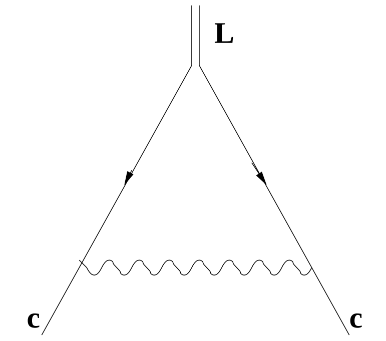

Figure 1. One-loop contribution to the *Lcc* vertex. The wavy lines represent the gluons, the straight line are for the ghosts.

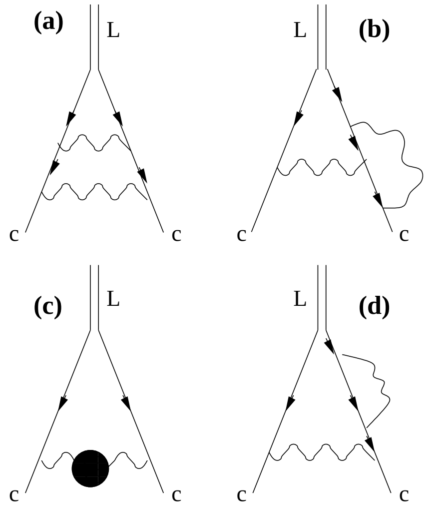

Figure 2. Two-loop diagrams for the *Lcc* vertex. The wavy lines represent the gluons, the straight lines the ghosts. The black disc in (c) is for one-loop contribution in the renormalization of the vector propagator from scalar, spinor and ghost fields.

to show this for the *Lcc* correlator, because other correlators can be expressed in terms of that one by ST identity, if we work in terms of dressed effective fields. The one-loop contribution shown in Fig. 1 (the only diagram that can be drawn) is apparently convergent in the Landau gauge. The point is that the derivatives can be integrated out of the graph due to the property of transversality of the gluon propagator in the Landau gauge that immediately makes this graph convergent in the ultraviolet region, but it is safe in the infrared. Note that in other gauges this correlator remains divergent in the ultraviolet, and its scale dependence is contained in the dressing function G_c. The two-loop diagrams (planar) are drawn below in Fig. 2. The first two diagrams are apparently convergent in ultraviolet in Landau gauge since all subgraphs are convergent. This is due to the property of transversality that again allows to integrate out the derivatives. The scale dependence in the diagram (c) cancels the scale dependence of diagram (d) because of $\mathcal{N} = 4$ supersymmetry (absence of the gauge coupling renormalization). Infrared region is also not dangerous since even in Landau gauge the gluon propagator is safe in the infrared region.

However, no invariant regularization scheme is known for supersymmetric theories, due to the well-known γ_5-problem: on general grounds finite renormalizations have to be recursively performed order by order in the loop expansion in order to preserve the relevant functional identities [26]. In this paragraph we discuss how one can treat this problem by using the technique described in this paper. The point is that the vertex Lcc is always convergent superficially in the Landau gauge. In $\mathcal{N}=4$ super-Yang–Mills theory ultraviolet divergences in the subgraphs of the Lcc vertex should cancel each other at the end. The insertion of the operator of the conformal anomaly into vacuum expectation values of operators of gluonic fields at different points in spacetime is proportional to the beta function of the gauge coupling [27]. Due to the algebra of the four-dimensional supersymmetry the beta function should be zero [14]. Algebra of the supersymmetry operators in the Hilbert space created by dressed fields can be considered as four-dimensional in Lorentz indices as well as in spinor indices since the limit $\epsilon \to 0$ is non-singular at one-loop order, two-loop order and higher orders as we have seen in the previous paragraphs. Thus, we can consider each correlator as pure four-dimensional, solving order-by-order the problem of dimensional discrepancy of convolutions in Lorentz and spinor indices.

The behavior of the theory in the IR region is not spoiled by the higher derivative regularization too. This is clear from the structure of the gluon propagator (in Landau gauge, for example) [3]:

$$D^{ab}_{\mu\nu} = \delta^{ab}\left[-\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\frac{1}{k^2 + k^6/\Lambda^4}\right].$$

Sixth degree of momentum in the denominator improves significantly ultraviolet behavior but in the infrared it is negligible in comparison with second degree of momentum. The infrared divergence appears when we en force put the on-shell condition $p_i^2 = 0$, where p_i are external momenta. In general, infrared region is not dangerous off shell in component formulation in Wess–Zumino gauge when we regularize the theory by higher derivatives.

In such way we come to our main conclusion in this paper. Namely, $\mathcal{N}=4$ supersymmetric theory has scale-independent effective action in terms of the dressed effective fields. All the dependence on the dimensionful parameter of ultraviolet regularization remains in the dressing functions only. This is in correspondence with direct calculation of anomalous dimensions and beta function in $\mathcal{N}=4$ theory [13]. At one-loop level, kernels of this scale-independent theory are in general dilogarithms in momentum space. These dilogarithms are Fourier transforms of the kernels in position space as given below. For example, the correlator of the dressed effective fields $\tilde{L}$, $\tilde{c}$, and $\tilde{c}$ at one loop level in any $SU(N)$ gauge theory has, among others, the following contribution:

$$\langle \tilde{L}^a(x)\tilde{c}^b(y)\tilde{c}^c(z)\rangle \sim g^2 N \frac{1}{((z-y)^2)^2(x-y)^2(z-x)^2} f^{abc}, \tag{8}$$

where the dressed effective fields are made of undressed effective fields convoluted to the dressing functions. The latter are unspecified but they are parts of the two point proper Green functions. The terms of the type (8) can be obtained, for example, by calculating the one-loop Lcc Green function in the Landau gauge (where $G_c(x) = \delta^{(4)}(x)$) and then using repeatedly the identity

$$\frac{1}{(2\pi)^4}\int d^4k\, \frac{e^{-ikx}}{k^2 + i\epsilon} = \lim_{\eta \to +0} \frac{i}{4\pi^2}\frac{1}{[(|x^0| - i\eta)^2 - \boldsymbol{x}^2]}.$$

By the ST identity the correlators (8) are related to the vertex KAc which in its turn is related to the tree gluon vertex AAA. The relation of these vertices is dictated by the ST identity and can be explicitly verified. Thus, the contribution similar to (8) can be found in the proper correlator of the three dressed gluons at one-loop level $\langle \tilde{A}^a_\mu(x)\tilde{A}^b_\nu(y)\tilde{A}^c_\lambda(z)\rangle$. At the same time, in $\mathcal{N}=4$ supersymmetry we do not need to make additional renormalization in two point gluon function to absorb the rest of infinities from it into renormalization of the gauge coupling, since the β function is zero.

We found that the dressed effective fields are the actual variables of the effective action. The effective action is to be written in terms of these dressed effective fields. In general, in non-supersymmetric gauge theory like QCD the dependence on UV regularization scale will be present inside the correlators of the dressed effective fields because it is necessary to remove the dependence on this scale by renormalization of the gauge coupling constant. In $\mathcal{N} = 4$ supersymmetric theory such a renormalization does not take place. Thus, the kernels for the dressed fields do not depend on scale. This might make possible an analysis of these kernels by tools of conformal field theory in all orders of perturbation theory.

We have shown in this paper that such a scale independent structure of correlators is a direct consequence of the Slavnov–Taylor identity and it is encoded in the Lcc correlator of the dressed effective fields. In general, by solving step-by-step the ST identity it is possible to reproduce structure of all n-gluonic proper correlators in terms of dressed effective fields. Here the question how to define the concept of scattering may arise. The knowledge of correlators is not sufficient to define a scattering matrix. Indeed, it follows by the above construction that these correlators of dressed mean fields in $\mathcal{N} = 4$ supersymmetry do not have dependence on any mass parameter, or, stated otherwise, the theory in terms of dressed mean fields is conformally invariant. It is known that S-matrix in conformal field theory cannot be constructed, since we do not have any dimensional parameter like mass or scale to define scattering concepts like typical scattering length, or size of a meson and so on. The only observables in this theory are correlators of the gauge invariant operators and their anomalous dimensions. However, on shell, when we go to the amplitudes, the scale appears due to infrared on shell divergences, so that scattering concepts can be introduced in a traditional way [28].

To conclude, in $\mathcal{N} = 4$ supersymmetric Yang–Mills theory there are two types of scale-independent correlators. They are, first, correlators of BPS gauge invariant operators which can be found from AdS/CFT correspondence and, second, correlators of the dressed mean fields, for example, dressed gluons in Landau gauge, that have been considered in this paper. Correlators of the gauge invariant operators are surely gauge independent. Non-BPS operators are scale-dependent and their anomalous dimensions are related to the infrared singularities of gluon amplitudes that are on-shell correlators in the momentum space [28]. As to correlators of the dressed mean fields, their gauge dependence is an open question. We make a conjecture that all gauge dependence is contained in the dressing functions of the dressed mean fields, and that the kernels of the dressed mean fields are gauge invariant.

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