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CORRIGENDUM TO "A NOTE ON WAVELETS AND S-ASYMPTOTICS"

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The paper **A Note on Wavelets and S–asymptotics**, by A. Takači and N. Teofanov, appeared in Matematički Vesnik, Vol. **49**, 3–4 (1997), 215–220. Unfortunately, due to an editorial error, an old version of the paper was printed. In order to correct this error, we give the correct end of the paper, starting from the eight line on page 218 (after the end of Proof of Theorem 1). We apologize to the authors and to the readers.

From now on we assume that the parameter a is greater than zero with no loss of generality. In order to give a theorem of Tauberian type, we rewrite the general Tauberian theorem from the paper: S. Pilipović and B. Stanković, *Tauberian theorems for integral transforms of distribution*, Acta Math. Hungar. **74**, 1–2 (1997), 135–153.

THEOREM 2. (Pilipović, Stanković) *Assume for $f \in \mathcal{D}'$ and $K \in C^\infty(\mathbf{R})$ the following conditions:*

a) The set $\left\{ \dfrac{f(x+h)}{c(h)};\ \ h \in \mathbf{R} \right\}$ *is bounded in* $\mathcal{D}'$.

b) There exists $\delta > 0$ such that

$$\eta(x)\check{K}(x)\exp((\alpha+\delta)x), \quad (1-\eta(x))\check{K}(x)\exp(\beta x) \in \mathcal{D}_{L^1} \quad \alpha > \beta,$$

where $\eta \in C^\infty(\mathbf{R})$ is equal to 1 in a neighbourhood of $+\infty$ and to 0 in a neighbourhood of $-\infty$, and $\mathcal{D}_{L^1}$ is a space of infinitely differentiable functions ϕ on $\mathbf{R}$ such that

$$\|\phi^{(n)}(x)\|_{L^1} < \infty, \ \ n \in \{0, 1, \dots\}.$$

c) $\hat{K}(\xi - i\alpha) \neq 0, \quad \xi \in \mathbf{R}$.

d) $\displaystyle\lim_{h\to\infty} \frac{(f*K)(h)}{c(h)} = C \int_{\mathbf{R}} \exp(-\alpha t)K(t)\,dt, \ \ C \in \mathbf{R}.$

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Then for every infinitely smooth function $\phi(x)$, $x \in \mathbf{R}$, satisfying

$$\eta(x)\check{\phi}(x)\exp((\alpha+\delta)x), \ \ (1-\eta(x))\check{\phi}(x)\exp(\beta x) \in \mathcal{D}_{L^1},$$

it holds

$$\lim_{h\to\infty} \frac{(f*\phi)(h)}{c(h)} = C\int_{\mathbf{R}} \exp(-\alpha t)\phi(t)\,dt.$$

THEOREM 3. *Let the analyzing wavelet ψ belong to $\mathcal{K}_1$, $\psi \not\equiv 0$, and assume that for every $\xi \in \mathbf{R}$ and for some $\alpha \in \mathbf{R}$ it holds*

$$\hat{\psi}(\xi - i\alpha) \neq 0.$$

Moreover, let T be a distribution from $\mathcal{K}_1'$ such that the set

$$\left\{\frac{T(x+h)}{c(h)},\ h \in \mathbf{R}\right\}$$

is bounded in $\mathcal{D}'$. If the wavelet transform $W_{\psi_a}T$ satisfies

$$\lim_{h\to\infty} \frac{(W_{\psi_a}T)(h)}{c(h)} = C\langle e^{-\alpha x}, \check{\psi}_a(x)\rangle, \quad C \in \mathbf{R} \tag{1}$$

then T has S–asymptotics, and we have

$$\lim_{h\to\infty}\left\langle\frac{T(x+h)}{c(h)}, \phi(x)\right\rangle = C\left\langle e^{-\alpha x}, \phi(x)\right\rangle, \quad (\phi \in \mathcal{K}_1).$$

The proof of Theorem 3 follows from Theorem 2, since the upper condition *b)* is satisfied for the elements of $\mathcal{K}_1$. Thus it holds

$$\begin{aligned}\lim_{h\to\infty}\left\langle\frac{T(x+h)}{c(h)}, \phi(x)\right\rangle &= \lim_{h\to\infty}\left\langle\frac{T(x)}{c(h)}, \phi(x-h)\right\rangle = \lim_{h\to\infty}\frac{(T*\check{\phi})(h)}{c(h)} \\ &= C\int_{\mathbf{R}} e^{-\alpha x}\check{\phi}(x)\,dx = C\left\langle e^{-\alpha x}, \phi(x)\right\rangle \quad (\forall\phi \in \mathcal{K}_1). \quad \blacksquare\end{aligned}$$

REMARKS. 1. Since $W_{\psi_a}T$ is a convolution between an element from $\mathcal{K}_1'$ and an infinitely differentiable function from $\mathcal{K}_1$ it holds $W_{\psi_a}T \in C^\infty \cap \mathcal{K}_1'$ and therefore the behavior given by (1) is in the ordinary sense.

2. The condition *c)* on the Fourier transform is satisfied for the Gaussian wavelets and also for the following one:

$$\psi(x) = e^{-x^2/2} - \frac{1}{2}e^{-x^2/8}.$$

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