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ON A CURVATURE TENSOR OF KÄHLER TYPE IN AN ALMOST HERMITIAN AND ALMOST PARA-HERMITIAN MANIFOLD

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Abstract. We find a tensor of Kähler type for an almost Hermitian and almost para-Hermitian manifold. We show that this tenosr is closely related with the problem of almost Hermitian and almost para-Hermitian manifold with pointwise constant holomorphic sectional curvature.

1. Preliminaries

Let (M, J, g) be an n $(= 2m)$-dimensional Riemannian manifold endowed with endomorphism J satisfying

$$J^2 = \varepsilon I, \qquad g(Jx, Jy) = -\varepsilon g(x, y), \tag{1.1}$$

where g is the metric of the manifold, $\varepsilon = \pm 1$, I indicates the identity mapping, $x, y \in T_p$ and T_p is the tangent space to M at $p \in M$. If $\varepsilon = -1$, (M, J, g) is an almost Hermitian manifold. If $\varepsilon = +1$, (M, J, g) is an almost para-Hermitian manifold [1]. In both cases

$$F(x, y) = g(Jx, y) \tag{1.2}$$

satisfies

$$F(x, y) = -F(y, x). \tag{1.3}$$

It is worth to mention that an almost para-Hermitian manifold is a semi-Riemannian manifold of signature (m, m).

In both cases, the vectors x and Jx are mutually orthogonal. This, in the case $\varepsilon = -1$ implies that x and Jx are linearly independent. In the case of almost para-Hermitian manifold, there exist vectors satisfying $Jx = x$ or $Jx = -x$. Such vector is a null vector and each null vector x satisfies $Jx = x$ or $Jx = -x$. For non-null vector x, x and Jx are linearly independent, too.

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2. Curvature tensor of Kähler type

We denote by ∇ and R the Riemannian connection and the curvature tensor of (M, J, g), respectively. An almost Hermitian (almost para-Hermitian) manifold satisfying $\nabla J = 0$ is a Kähler (para-Kähler) manifold. It is well known that the Riemannian curvature tensor of a Kähler (para-Kähler) manifold satisfies the condition

$$R(x,y,z,w) = -\varepsilon R(x,y,Jz,Jw). \tag{2.1}$$

Thus, we say that, in an almost Hermitian (almost para-Hemritian) manifold, the tensor $Q(x,y,z,w)$ is a curvature tensor of Kähler type, if it satisfies the conditions

$$Q(x,y,z,w) = -Q(x,y,w,z), \tag{2.2}$$
$$Q(x,y,z,w) = -Q(y,x,z,w), \tag{2.3}$$
$$Q(x,y,z,w) = Q(z,w,x,y), \tag{2.4}$$
$$Q(x,y,z,w) + Q(x,z,w,y) + Q(x,w,y,z) = 0, \tag{2.5}$$
$$Q(x,y,z,w) = -\varepsilon Q(x,y,Jz,Jw). \tag{2.6}$$

For a general almost Hermitian (almost para-Hemritian) manifold, the Riemannian curvature tensor $R(x,y,z,w)$ does not satisfy (2.6). Thus, we shall try to construct a new tensor satisfying (2.2)–(2.6). The construction that follows is inspired by the paper [4].

We start with the tensor

$$\begin{aligned} &H(x,y,z,w) = \\ &\quad = 4[R(x,y,z,w) - \varepsilon R(x,y,Jz,Jw) - \varepsilon R(Jx,Jy,z,w) + R(Jx,Jy,Jz,Jw)]. \end{aligned} \tag{2.7}$$

It is easy to see that this tensor satisfies identities (2.2), (2.3), (2.4) and (2.6). If it satisfies (2.5), too, it must be

$$\begin{aligned} &R(x,y,Jz,Jw) + R(Jx,Jy,z,w) + R(x,z,Jw,Jy) + \\ &\qquad + R(Jx,Jz,w,y) + R(x,w,Jy,Jz) + R(Jx,Jw,y,z) = 0. \end{aligned} \tag{2.8}$$

Putting into (2.8) Jx and Jy instead of x and y, respectively, and taking into account (1.1), we get

$$\begin{aligned} &R(Jx,Jy,Jz,Jw) + R(x,y,z,w) + \varepsilon R(Jx,z,Jw,y) + \\ &\qquad + \varepsilon R(x,Jz,w,Jy) + \varepsilon R(Jx,w,y,Jz) + \varepsilon R(x,Jw,Jy,z) = 0. \end{aligned} \tag{2.9}$$

Subtracting (2.8) (multiplied by ε) from (2.9), we find

$$\begin{aligned} &R(x,y,z,w) - \varepsilon R(x,y,Jz,Jw) - \varepsilon R(Jx,Jy,z,w) + R(Jx,Jy,Jz,Jw) = \\ &\quad = \varepsilon[R(Jx,Jz,w,y) + R(x,w,Jy,Jz) + R(x,z,Jw,Jy) + R(Jx,Jw,y,z) \\ &\qquad - R(Jx,z,Jw,y) - R(x,Jz,w,Jy) - R(Jx,w,y,Jz) - R(x,Jw,Jy,z)]. \end{aligned}$$

Substituting this into (2.7) we obtain

$$\begin{aligned}
&H(x,y,z,w)=\\
&\quad=3[R(x,y,z,w)-\varepsilon R(x,y,Jz,Jw)-\varepsilon R(Jx,Jy,z,w)+R(Jx,Jy,Jz,Jw)]\\
&\qquad+\varepsilon[R(x,z,Jw,Jy)+R(Jx,Jz,w,y)+R(x,w,Jy,Jz)+R(Jx,Jw,y,z)\\
&\quad-R(Jx,z,Jw,y)-R(x,Jz,w,Jy)-R(Jx,w,y,Jz)-R(x,Jw,Jy,z)]
\end{aligned}\tag{2.10}$$

We can verify, by direct computation, that the tensor (2.10) satisfies all identities (2.2)–(2.6), that is, it is a curvature tensor of Kähler type.

If (M,J,g) is a Kähler (para-Kähler) manifold, then (2.1) holds good, because of which we have also

$$\begin{aligned}
R(x,y,z,w)&=-\varepsilon R(Jx,Jy,z,w),\\
R(x,y,z,w)&=R(Jx,Jy,Jz,Jw),\\
R(x,Jy,Jz,w)&=-R(x,Jy,z,Jw),\\
R(x,Jy,Jz,w)&=-R(Jx,y,Jz,w).
\end{aligned}\tag{2.11}$$

Therefore, (2.10) reduces to

$$H(x,y,z,w)=14R(x,y,z,w)-2\varepsilon[R(x,Jz,w,Jy)+R(x,Jw,Jy,z)].\tag{2.12}$$

But, in view of Bianchi identity and (2.11),

$$\begin{aligned}
&R(x,Jz,w,Jy)+R(x,Jw,Jy,z)=\\
&\quad=-R(x,w,Jy,Jz)-R(x,Jy,Jz,w)-R(x,Jy,z,Jw)-R(x,z,Jw,Jy)\\
&\qquad=\varepsilon[R(x,w,y,z)+R(x,z,w,y)]-R(x,Jy,Jz,w)+R(x,Jy,Jz,w)\\
&\qquad\qquad=-\varepsilon R(x,y,z,w).
\end{aligned}$$

This means that (2.12) reduces to $H(x,y,z,w)=16R(x,y,z,w)$. Thus, we can state

THEOREM 1. *For an almost Hermitian (almost para-Hermitian) manifold (M,J,g), the tensor*

$$\begin{aligned}
&A(x,y,z,w)=\\
&=\frac{1}{16}\{3[R(x,y,z,w)-\varepsilon R(x,y,Jz,Jw)-\varepsilon R(Jx,Jy,z,w)+R(Jx,Jy,Jz,Jw)]\\
&\qquad+\varepsilon[R(x,z,Jw,Jy)+R(Jx,Jz,w,y)+R(x,w,Jy,Jz)+R(Jx,Jw,y,z)\\
&\quad-R(Jx,z,Jw,y)-R(x,Jz,w,Jy)-R(Jx,w,y,Jz)-R(x,Jw,Jy,z)]\}
\end{aligned}\tag{2.13}$$

is a curvature tensor of Kähler type. If (M,J,g) is a Kähler (para-Kähler) manifold, (2.13) reduces to the Riemannian curvature tensor.

We note that, beside (2.2)–(2.6) the tensor (2.13) satisfies the identities of type (2.11), too, i.e.

$$\begin{aligned}
A(x,y,z,w)&=-\varepsilon A(Jx,Jy,z,w),\\
A(x,y,z,w)&=A(Jx,Jy,Jz,Jw),\\
A(x,Jy,Jz,w)&=-A(x,Jy,z,Jw),\\
A(x,Jy,Jz,w)&=-A(Jx,y,Jz,w).
\end{aligned}\tag{2.14}$$

3. The Ricci tensor with respect to the tensor (2.13)

Since we have $R(x,y,z,w) = g(R(z,w)y,x) = -\varepsilon g(JR(z,w)y,Jx)$, we have also

$$\begin{aligned} R(Jx,Jy,z,w) &= -g(JR(z,w)Jy,x),\\ R(Jx,Jy,Jz,Jw) &= -g(JR(Jz,Jw)Jy,x),\\ R(Jx,z,Jw,y) &= -g(JR(Jw,y)z,x),\\ R(Jx,w,y,Jz) &= -g(JR(y,Jz)w,x). \end{aligned}$$

Therefore, if the tensor $A(z,w)y$ is defined by $A(x,y,z,w) = g(A(z,w)y,x)$, we can rewrite (2.13) in the form

$$\begin{aligned} A(z,w)y = \frac{1}{16}\{&3[R(z,w)y - \varepsilon R(Jz,Jw)y + \varepsilon J(R(z,w)Jy) - J(R(Jz,Jw)Jy)]\\ &+ \varepsilon[R(Jw,Jy)z - J(R(w,y)Jz) + R(Jy,Jz)w - J(R(y,z)Jw)\\ &+ J(R(Jw,y)z) - R(w,Jy)Jz + J(R(y,Jz)w) - R(Jy,z)Jw]\}. \end{aligned} \tag{3.1}$$

We recall that the Ricci tensor $\rho(x,y)$ and the Hermitian Ricci tensor $\overset{*}{\rho}(x,y)$ of (M,J,g) are defined by

$$\rho(y,w) = \text{trace}\colon z \to R(z,w)y = \text{trace}\colon z \to \varepsilon J(R(Jz,w)y),$$

$$\overset{*}{\rho}(y,w) = \text{trace}\colon z \to R(Jz,w)y = \text{trace}\colon z \to J(R(z,w)y),$$

and are symmetric and skew-symmetric respectively, i.e.

$$\rho(y,w) = \rho(w,y), \qquad \overset{*}{\rho}(y,w) = -\overset{*}{\rho}(w,y). \tag{3.2}$$

Now, if we define $A(y,w)$ by $A(y,w) = \text{trace}\colon z \to A(z,w)y$, then we get from (3.1):

$$\begin{aligned} A(y,w) = \frac{1}{16}\{&3[\rho(y,w) - \varepsilon\overset{*}{\rho}(y,Jw) - \varepsilon\overset{*}{\rho}(w,Jy) - \varepsilon\rho(Jy,Jw)]\\ &+ \varepsilon[2\rho(Jw,Jy) - \rho(Jy,Jw) - 2\varepsilon\rho(w,y) + \varepsilon\rho(y,w)\\ &- 2\overset{*}{\rho}(w,Jy) + 2\overset{*}{\rho}(Jw,y) - \overset{*}{\rho}(y,Jw) + \overset{*}{\rho}(Jy,w)]\}, \end{aligned}$$

i.e.

$$A(y,w) = \frac{1}{8}[\rho(y,w) - 3\varepsilon\overset{*}{\rho}(y,Jw) - 3\varepsilon\overset{*}{\rho}(w,Jy) - \varepsilon\rho(Jy,Jw)]. \tag{3.3}$$

If (M,J,g) is a Kähler (para-Kähler) manifold, then $\rho(y,w) = -\varepsilon\rho(Jy,Jw) = -\varepsilon\overset{*}{\rho}(y,Jw) = -\varepsilon\overset{*}{\rho}(w,Jy)$, and (3.3) reduces to $A(y,w) = \rho(y,w)$. Thus, we have

THEOREM 2. *The Ricci tensor with respect to the tensor $A(x,y,z,w)$ is given by (3.3), and if (M,J,g) is a Kähler (para-Kähler) manifold, it reduces to the Ricci tensor $\rho(y,w)$.*

It is easy to see that the tensor $A(y,w)$ has the properties like the Ricci tensor $\rho(y,w)$ of a Kähler (para-Kähler) manifold, namely

$$A(x,y) = A(y,x), \ \ A(x,y) = -\varepsilon A(Jx,Jy), \ \ A(x,Jy) = -A(Jx,y),$$

$$A(x,y) = -\varepsilon\overset{*}{A}(x,Jy), \ \ \overset{*}{A}(x,y) = -\overset{*}{A}(y,x),$$

where $\overset{*}{A}(x,y) = \text{trace}\colon z \to A(Jz,y)x$.

We define the scalar curvature with respect to the curvature tensor $A(x,y,z,w)$ by

$$A = \text{trace}\colon w \to A(w), \tag{3.4}$$

where

$$A(y,w) = g(y, A(w)). \tag{3.5}$$

To obtain the expression for this scalar curvature, we consider the vectors $\rho(w)$ and $\overset{*}{\rho}(w)$ defined by

$$\begin{aligned} \rho(y,w) &= g(y,\rho(w)) = -\varepsilon g(Jy, J\rho(w)),\\ \overset{*}{\rho}(y,w) &= g(y,\overset{*}{\rho}(w)) = -\varepsilon g(Jy, J\overset{*}{\rho}(w)), \end{aligned} \tag{3.6}$$

and note that the scalar curvature τ and the Hermitian scalar curvature $\overset{*}{\tau}$ can be defined by

$$\begin{aligned} \tau &= \text{trace}\colon w \to \rho(w) = \text{trace}\colon w \to \varepsilon J\rho(Jw),\\ \overset{*}{\tau} &= \text{trace}\colon w \to \overset{*}{\rho}(w) = \text{trace}\colon w \to J\overset{*}{\rho}(w). \end{aligned} \tag{3.7}$$

Using (3.5) and (3.6), we can rewrite (3.3) in the form

$$A(w) = \frac{1}{8}[\rho(w) - 3\varepsilon\overset{*}{\rho}(Jw) - 3\varepsilon J\overset{*}{\rho}(w) + \varepsilon J\rho(Jw)],$$

from which, in view of (3.4) and (3.7), we get

$$A = \frac{1}{4}(\tau - 3\varepsilon\overset{*}{\tau}). \tag{3.8}$$

If (M,J,g) is a Kähler (para-Kähler) manifold, then $\overset{*}{\tau} = -\varepsilon\tau$ and (3.8) reduces to $A = \tau$. Thus, we can state

THEOREM 3. *The scalar curvature with respect to the tensor $A(x,y,z,w)$ is given by (3.8). If (M,J,g) is a Kähler (para-Kähler) manifold, it reduces to the scalar curvature τ with respect to the Riemannian curvature tensor.*

4. Almost Hermitian and almost para-Hermitian manifold with pointwise constant holomorphic sectional curvature

As is well known, the holomprphic sectional curvature of (M,J,g) at $p \in M$, determined by non-null vector $x \in T_p$, is given by

$$\frac{R(x,Jx,x,Jx)}{g(x,x)g(Jx,Jx) - g(x,Jx)g(x,Jx)} = -c(p),$$

i.e. by

$$\frac{R(x,Jx,x,Jx)}{-\varepsilon g(x,x)g(x,x)} = -c(p) \tag{4.1}$$

because of (1.1) and the orthogonality of vectors x and Jx.

If the scalar $c(p)$ is independent of the choice of vector $x \in T_p$, then (M,J,g) is said to be a manifold of pointwise constant holomorphic sectional curvature at $p \in M$. The almost Hermitian manifolds of pointwise constant holomorphic sectional curvature have been studied by many authors ([2], [5], [6], [7], [8]). The purpose of this section is to show that some of these questions (parallely for almost Hermitian and almost para-Hermitian manifolds) can be considered with the help of the tensor (2.13).

We define the sectional curvature at the point $p \in M$, with respect to the tensor $A(x,y,z,w)$, determined by non-null vectors $x, y \in T_p$, as follows

$$\frac{A(x,y,x,y)}{g(x,x)g(y,y)-g(x,y)g(x,y)} = -c(p).$$

If $y = Jx$, we have the holomorphic sectional curvature with respect to $A(x,y,z,w)$:

$$\frac{A(x,Jx,x,Jx)}{-\varepsilon g(x,x)g(x,x)} = -c(p). \tag{4.2}$$

On the other hand, putting in (2.13) $y = Jx$, $z = x$, $w = Jx$, we find $A(x,Jx,x,Jx) = R(x,Jx,x,Jx)$. Thus, (M,J,g) is a manifold of pointwise constant holomorphic sectional curvature if and only if it has this property with respect to the tensor $A(x,y,z,w)$, too. But, $A(x,y,z,w)$ is a curvature tensor of Kähler type and we can proceed with it in the same manner as with the Riemannian curvature tensor in the case of Kähler space, e.g. [9], p. 75 or [3], pp. 165–168. Namely, we rewrite (4.2) in the form

$$A(x,Jx,x,Jx) - \varepsilon c(p)g(x,x)g(x,x) = 0, \tag{4.3}$$

and consider the quadrilinear mapping which sends $(x,y,z,w) \in T_p \otimes T_p \otimes T_p \otimes T_p$ into

$$\begin{aligned} &A(Jx,y,Jz,w) + A(Jy,z,Jx,w) + A(Jz,x,Jy,w) \\ &\qquad - \varepsilon c(p)[g(x,y)g(z,w) + g(y,z)g(x,w) + g(x,z)g(y,w)]. \end{aligned} \tag{4.4}$$

This mapping is symmetric in x, y, z and w because $A(x,y,z,w)$ satisfies the identities (2.2), (2.3), (2.4) and (2.6). For $x = y = z = w$, (4.4) reduces to $3A(Jx,x,Jx,x) - 3\varepsilon c(p)g(x,x)g(x,x)$. But this vanishes by the assumption (4.3). Thus, the mapping (4.4) vanishes identically, i.e.

$$\begin{aligned} &A(Jx,y,Jz,w) + A(Jy,z,Jx,w) + A(Jz,x,Jy,w) = \\ &\qquad = \varepsilon c(p)[g(x,y)g(z,w) + g(y,z)g(x,w) + g(x,z)g(y,w)]. \end{aligned} \tag{4.5}$$

Putting into (4.5) Jx and Jy instead of x and y respectively, and taking into account (1.1) and (2.14), we have

$$\begin{aligned} &A(x,y,z,w) - A(y,z,x,w) + \varepsilon A(z,Jx,Jy,w) = \\ &\qquad = \varepsilon c(p)[g(Jx,y)g(Jz,w) + g(y,Jz)g(Jx,w) - \varepsilon g(x,z)g(y,w)]. \end{aligned}$$

Taking the skew-symmetric part of this equation with respect to x and y, we obtain

$$\begin{aligned} &2A(x,y,z,w) - A(y,z,x,w) + A(x,z,y,w) + \varepsilon[A(z,Jx,Jy,w) - A(z,Jy,Jx,w)] = \\ &\qquad = \varepsilon c(p)\{2g(Jx,y)g(Jz,w) + g(y,Jz)g(Jx,w) - g(x,Jz)g(Jy,w) \\ &\qquad\qquad - \varepsilon[g(x,z)g(y,w) - g(x,w)g(y,z)]\}. \end{aligned} \tag{4.6}$$

In view of (2.2)–(2.6), we have $-A(y,z,x,w) + A(x,z,y,w) = A(x,y,z,w)$ and

$$\varepsilon[A(z,Jx,Jy,w) - A(z,Jy,Jx,w)] = -\varepsilon A(z,w,Jx,Jy) = A(x,y,z,w).$$

Therefore, and taking into account (1.2), we can rewrite (4.6) as follows:

$$\begin{aligned} A(x,y,z,w) = \frac{c(p)}{4}\{&g(x,w)g(y,z) - g(x,z)g(y,w) \\ &+ \varepsilon[-F(x,w)F(y,z) + F(x,z)F(y,w) + 2F(x,y)F(z,w)]\}. \end{aligned} \tag{4.7}$$

Thus, we can state

THEOREM 4. *An almost Hermitian (almost para-Hermitian) manifold is a manifold with a pointwise constant holomorhic sectional curvature if and only if the tensor $A(x,y,z,w)$ has the form (4.7).*

REMARK. We obtain the same result dealing directly with (4.1). Namely, we rewrite (4.1) in the form

$$R(x,Jx,x,Jx) - \varepsilon c(p)g(x,x)g(x,x) = 0 \tag{4.8}$$

and consider the quadrilinear mapping which sends $(x,y,z,w) \in T_p \otimes T_p \otimes T_p \otimes T_p$ into

$$\begin{aligned} \frac{1}{16}\{&R(Jx,y,Jw,z) + R(Jx,z,Jw,y) + R(Jx,w,Jz,y) + R(Jx,y,Jz,w) \\ &+ R(Jx,z,Jy,w) + R(Jx,w,Jy,z) + R(Jy,w,Jz,x) + R(Jy,x,Jz,w) \\ &+ R(Jy,x,Jw,z) + R(Jy,z,Jw,x) + R(Jw,y,Jz,x) + R(Jw,x,Jz,y) \\ &- 4\varepsilon c(p)[g(x,y)g(w,z) + g(x,w)g(z,y) + g(x,z)g(y,w)]\}. \end{aligned}$$

This mapping is symmetric in x, y, z and w and since it vanishes for $x = y = z = w$ by the assumption (4.8), it must vanish identically, i.e.

$$\begin{aligned} \frac{1}{16}[&R(Jx,y,Jw,z) + R(Jx,z,Jw,y) + R(Jx,w,Jz,y) + R(Jx,y,Jz,w) \\ &+ R(Jx,z,Jy,w) + R(Jx,w,Jy,z) + R(Jy,w,Jz,x) + R(Jy,x,Jz,w) \\ &+ R(Jy,x,Jw,z) + R(Jy,z,Jw,x) + R(Jw,y,Jz,x) + R(Jw,x,Jz,y)] = \\ &= \varepsilon\frac{c(p)}{4}[g(x,y)g(w,z) + g(x,w)g(z,y) + g(x,z)g(y,w)]. \end{aligned} \tag{4.9}$$

Now, proceeding with (4.9) in a similar manner as with (4.5), we get (4.7).

5. Sato's form of the tensor $A(x,y,z,w)$

Sato proved ([5], Theorem 4.2) that the curvature tensor of an almost Hermitian manifold of pointwise constant holomorphic sectional curvature $c(p)$ is given by

$$\begin{aligned} R(x,y,z,w) &= \frac{c(p)}{4}[g(x,w)g(y,z) - g(x,z)g(y,w) \\ &\qquad + F(x,w)F(y,z) - F(x,z)F(y,w) - 2F(x,y)F(z,w)] \\ &= \frac{1}{96}\{26[G(x,y,z,w) - G(z,w,x,y)] - 6[G(Jx,Jy,Jz,Jw) + G(Jz,Jw,Jx,Jy)] \\ &\qquad + 13[G(x,z,y,w) + G(y,w,x,z) - G(x,w,y,z) - G(y,z,x,w)] \\ &\qquad - 3[G(Jx,Jz,Jy,Jw) + G(Jy,Jw,Jx,Jz) - G(Jx,Jw,Jy,Jz) - G(Jy,Jz,Jx,Jw)] \\ &\qquad + 4[G(x,Jy,z,Jw) + G(Jx,y,Jz,w)] \\ &\qquad + 2[G(x,Jz,y,Jw) + G(Jx,z,Jy,w) - G(x,Jw,y,Jz) - G(Jx,w,Jy,z)]\}, \end{aligned}$$

where $G(x,y,z,w) = R(x,y,z,w) - R(x,y,Jz,Jw)$.

To comprise the almost para-Hermitian manifold, too, we shall consider the tensor

$$R(x,y,z,w) = \frac{1}{96}\{26[G(x,y,z,w) - G(z,w,x,y)] - 6[G(Jx,Jy,Jz,Jw) + G(Jz,Jw,Jx,Jy)] + 13[G(x,z,y,w) + G(y,w,x,z) - G(x,w,y,z) - G(y,z,x,w)] - 3[G(Jx,Jz,Jy,Jw) + G(Jy,Jw,Jx,Jz) - G(Jx,Jw,Jy,Jz) - G(Jy,Jz,Jx,Jw)] - 4\varepsilon[G(x,Jy,z,Jw) + G(Jx,y,Jz,w)] - 2\varepsilon[G(x,Jz,y,Jw) + G(Jx,z,Jy,w) - G(x,Jw,y,Jz) - G(Jx,w,Jy,z)]\}, \quad (5.1)$$

where

$$G(x,y,z,w) = R(x,y,z,w) + \varepsilon R(x,y,Jz,Jw). \quad (5.2)$$

Substituting (5.2) into (5.1), we obtain, after some simple but long computation, the expression on right-hand side of the relation (2.13). Thus, we can say that (5.1) is Sato's form of the tensor $A(x,y,z,w)$. Also, we see that, for an almost Hermitian manifold, Theorem 4 coincides with mentioned Sato's theorem.

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