Journal of Integer Sequences, Vol. 18 (2015),
Article 15.4.3

Space-Efficient Generation of Nonisomorphic Maps and Hypermaps

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Abstract

In 1979, while working as a senior researcher in the Computing Centre of the USSR Academy of Sciences in Moscow, I used Lehman's code for rooted maps of any orientable genus to generate these maps. By imposing an order on the code-words and keeping only those that are maximal over all the words that code the same map with each semi-edge chosen as the root, I generated these maps up to orientation-preserving isomorphism, and by comparing each of them with the code-words for the map obtained by reversing the orientation, I generated these maps up to a generalized isomorphism that could be orientation-preserving or orientation-reversing. The limitations on the speed of the computer I was using and the time allowed for a run restricted me to generating these maps with up to only six edges. In 2011, by optimizing the algorithms and using a more powerful computer and more CPU time I was able to generate these maps with up to eleven edges. An average-case time-complexity analysis of the generation algorithms is included in this article. And now, by using a genus-preserving bijection between hypermaps and bicoloured bipartite maps that I discovered in 1975 and the condition on the word coding a rooted map for the map to be bipartite, I generated hypermaps, both rooted and unrooted, with up to twelve darts (edge-vertex incidence pairs).

1 Introduction

A *map* is defined topologically as a 2-cell embedding [3] of a connected graph, loops and multiple edges allowed, in a 2-dimensional surface. The *faces* of a map are the connected components of the complement of the graph in the surface. In this article the surface is assumed to be without boundary and orientable, with an orientation already attributed to it (counter-clockwise, say), so that it is completely described by a non-negative integer g, its *genus*. For short, a map on a surface of genus g will be called a *map of genus* g. A *planar map* is a map of genus 0 (a map on a sphere). If a map on a surface of genus g has v vertices, e edges and f faces, then by the Euler-Poincaré formula [8, Chap. 9]

$$f - e + v = 2(1 - g). \tag{1}$$

Two maps are *equivalent* if there is an orientation-preserving homeomorphism between their embedding surfaces that takes the vertices, edges and faces of one map into the vertices, edges and faces of the other. A *dart* or *semi-edge* of a map or graph is half an edge. A loop is assumed to be incident twice to the same vertex, so that every edge, whether or not it is a loop, contains two darts. The face incident to a dart d is the face incident to the edge containing d and on the right of an observer on d facing away from the vertex incident to d. A *rooted map* is a map with a distinguished dart, its *root*. Two rooted maps are equivalent if there is an orientation-preserving homeomorphism between their embedding surfaces that takes the vertices, edges, faces and the root of one map into the vertices, edges, faces and the root of the other.

A *combinatorial map* is a connected graph with a cyclic order imposed on the darts incident to each vertex, representing the order in which the darts of a (topological) map are encountered during a rotation around the vertex according to the orientation of the embedding surface. Given a dart d, we denote by $-d$ the other half of the edge containing d and by $P(d)$ the next dart after d according to the cyclic order of the darts around the vertex incident to d. The darts incident to a face are encountered by successive application of the permutation $P-$ ($-$ followed by P). In this way the faces of a combinatorial map can be counted, so that its genus can be calculated from (1). Two combinatorial maps are equivalent if they are related by a *map isomorphism* – a graph isomorphism that preserves this cyclic order – with an analogous definition for the equivalence of two rooted combinatorial maps. An *automorphism* of a combinatorial map is a map isomorphism from a map onto itself.

Following [36] and [35], we define a *sensed map* to be an equivalence class of maps and an *unsensed map* to be an equivalence class of maps under a homeomorphism that could be orientation-preserving or orientation-reversing. It was shown in [14] that each equivalence class of topological maps is uniquely defined by an equivalence class of combinatorial maps; so from now on a rooted map means a rooted combinatorial map, a sensed map means an isomorphism class of combinatorial maps and an unsensed map means an equivalence class of maps under both isomorphism and reversal of the cyclic order imposed on the darts incident to each vertex.

Map enumeration began in earnest with the work of Tutte, who used it in an attempt to solve the famous four-colour problem. Lehman used it in his study of the molecular structure of polymers. In addition, map enumeration has applications in classical and algebraic combinatorics [11], theoretical physics and integrable hierarchies [15].

There are many research papers on the enumeration of maps with various properties; we list here some of the papers in which maps (rooted, sensed and unsensed) have been enumerated by genus and either number of edges alone or number of edges and vertices (the latter is equivalent by (1) to enumerating by number of faces and vertices).

Rooted planar maps were counted by Tutte, first by number of edges alone (as a closed-form formula) [24] and then by number of faces and vertices (as a generating function) [25]. I found an algorithm for counting rooted maps by genus, number of edges and number of vertices [27, 34] and a polynomial algorithm for counting rooted *toroidal* maps (maps of genus 1), both by number of edges and by number of vertices and faces [30]. Using an improved version of the method of [27], presented by Bender and Canfield [4], Arquès found a closed-form formula for counting rooted toroidal maps, both by number of edges and by number of vertices and faces [3]. Bender and Canfield found a closed-form formula for counting rooted maps of genus 2 and 3 by number of edges [5]. Giorgetti, a student of Arquès, generalized the results of [3] and [5] to obtain a general form for the generating function counting rooted maps of any genus by number of vertices and faces and counted the maps of genus 2 and 3 [9]. I then collaborated with Giorgetti to extend this enumeration up to genus 6 [32] and later up to genus 10 [33].

Liskovets found a closed-form formula for the number of sensed planar maps by number of edges [16]. Mednykh and Nedela generalized Liskovets' method and thus counted sensed maps of genus 1, 2 and 3 by number of edges [18] and then Giorgetti and Mednykh counted sensed maps of genus 4 by number of edges [17]. Then I collaborated with Giorgetti and Mednykh to count sensed maps of genus up to 10 by number of vertices and faces and up to genus 11 by number of edges [33, 31]. Using a more efficient method for counting rooted maps discovered by Carrell and Chapuy [6], Giorgetti and I enumerated rooted and sensed maps of genus up to 50 with up to 100 edges in [10], which includes tables of numbers of sensed maps of genus up to 19. And Wormald found an algorithm for counting planar maps, both sensed and unsensed, by number of edges and by number of vertices and faces [36, 35]. The methods used to obtain all of the above results are computationally more efficient than exhaustive generation. But, as far as I know, exhaustive generation is the only method yet known to enumerate unsensed non-planar maps, and even for maps that have been enumerated by other methods, exhaustive generation serves to verify the numbers obtained by these methods.

The method I used in [29] to generate isomorphism classes of maps without having to store all the previously generated rooted maps to see whether each new map is isomorphic to one of the old ones is essentially the one used by Read [20] to generate the isomorphism classes of 9-vertex graphs. He generated the adjacency matrix of each of the labelled 9-vertex graphs and then eliminated all those that are not lexicographically largest among those matrices representing the same graph but with a different labelling. Since a rooted map has only the trivial automorphism [24], I generated all the rooted maps, or rather Lehman's code for rooted maps, with e edges and v vertices, eliminated all those whose code-word is not lexicographically largest among those coding the same map but with a different root, and sorted the rest by genus to generate sensed maps; to generate unsensed maps, I eliminated each sensed map whose code-word could be made lexicographically larger by reversing the cyclic order of the darts at each vertex and choosing one of the darts as the root. To be sure, more sophisticated methods of generating isomorphism classes of combinatorial objects have since been discovered [12], and for objects with many distinct labellings these methods are probably much faster. However, a map with e edges has at most $2e$ distinct rootings; so the admittedly old-fashioned method I used seems to be quite adequate.

More recently Jackson and Visentin published an atlas of maps [13].

A (combinatorial) *hypermap* is a generalization of a map in which an edge is allowed to have any positive number of darts instead of exactly two and the darts are cyclically ordered around the edges as well as the vertices. In 1975 I published a genus-preserving bijection between hypermaps with d darts, e edges, v vertices and f faces and bicoloured bipartite maps with d edges, e black vertices, v white vertices and f faces, each containing twice as many darts as the corresponding face of the hypermap [28]. This bijection was used by Arquès to count rooted planar [1] and toroidal [2] hypermaps by number of vertices, edges and faces; Chauve [7] independently counted rooted bicoloured bipartite planar maps with the corresponding parameters. And now I discovered a condition on the Lehman word that codes a rooted map for the map to be bipartite, which I used to generate rooted, sensed and unsensed hypermaps with up to 12 darts.

The words with which Lehman coded rooted maps are described in Section 2, the procedure I used to generate these words is described and analyzed in Section 3, a discussion of the generation of hypermaps appears in Section 4 and the results of the computation, including timings, are described in Section 5. A table of numbers of unsensed maps with up to 11 edges, sorted by genus and number of vertices, appears in Appendix 6; the analogous tables for rooted maps and sensed maps appear in other sources, which are cited in Section 5. Appendix 6 contains a table of numbers of rooted, sensed and unsensed hypermaps.

2 Lehman's code for rooted maps

In the 1960s Lehman, who was then my Ph. D. supervisor, generalized the code for a rooted plane tree as a balanced parenthesis system to a code for a rooted planar map with a given spanning tree as a (balanced) parenthesis system (coding the rooted plane tree obtained by deleting the edges not in the spanning tree) merged with a bracket system (coding the rooted one-vertex map obtained by contracting the edges of the spanning tree). The number of pairs of parentheses is the number of edges of the spanning tree and the number of pairs of brackets is the number of edges not in the spanning tree. To code a rooted planar map without a spanning tree, he used Tamari's maze-running algorithm [23], which is essentially depth-first search [22] with the darts incident to each vertex encountered in their cyclic order, to construct a canonical spanning tree, and he proved that a spanning tree is canonical if and only if the code word for the rooted map with this spanning tree does not contain the forbidden sub-word [(]), where the right bracket is the *mate* of (that is, closes) the left bracket, the right parenthesis is the mate of the left parenthesis and the four symbols are not necessarily contiguous.

To code a rooted map of any orientable genus, he replaced the bracket system by an *integer system on m pairs*: a word consisting of two copies of each of the integers 1, 2, ..., m, where m is the number of edges in the rooted one-vertex map coded by the integer system and the first occurrences of the integers are in increasing order. The forbidden sub-word is now $i(i)$, where the right parenthesis is the mate the left one.

Each letter in a word coding a rooted map represents a dart, with the first letter representing the root. If a dart d is (represented by) a parenthesis or a bracket, then $-d$ is its mate; if d is an integer i, then $-d$ is the other occurrence of i. If d is a bracket or an integer, then $P(d)$ is the next letter in the word (with wraparound); if d is a parenthesis, then $P(d)$ is the letter that follows the mate of d (with wraparound). The darts of the face containing d can be found from the code-word by successive application of the permutation $P-$ to the letters representing the darts. For example, in the code word 123123, the face containing the first 1 also contains the second 2 and the first 3 (the next dart would be the first 1) and the face containing the first 2 also contains the second 3 and the second 1; since all the letters belong to one of these two faces, there are only 2 faces and so by (1) the one-vertex map coded by this word is of genus 1. Since contracting an edge does not change the genus of a map, the genus of a rooted map can be calculated from the integer sub-system of its code-word.

A more detailed description of Professor Lehman's code, including his method of coding a rooted map, can be found in my Ph. D. thesis [27] and in [29], where I described the use I made of his code to generate isomorphism classes of maps.

3 Generating maps

To generate the rooted plane trees with e edges, I generate the parenthesis systems with e pairs of parentheses in lexicographical order, with a left parenthesis represented by 0 and a right parenthesis represented by -1. To generate the rooted planar one-vertex maps with e edges, I generate the bracket systems with e pairs of brackets, also in lexicographical order, with a left bracket represented by 2 and a right bracket represented by 1. To generate the not-necessarily-planar rooted one-vertex maps with e edges, I generate the integer systems on e pairs; in [29] I made the second occurrence of each integer move from its leftmost position (immediately to the right of the first occurrence of the same integer) to its rightmost position (the rightmost letter in the word) with e moving the fastest, whereas now I use a Gray code in which they move alternately to the right and to the left. Each new system is generated in $O(e)$ time in the worst case and $O(1)$ time in the average case.

To generate the rooted maps with e edges and v vertices, I first generate the bracket systems or the integer systems on e-v+1 pairs, and in the latter case I calculate the genus by counting the faces (in $O(e)$ time) and substituting into (1) as described above. For each bracket system or integer system I generate all the parenthesis systems on $v-1$ pairs. For each pair of words I merge them in all possible ways that avoid the forbidden sub-word, moving each parenthesis from left to right, with a right parenthesis starting adjacent to its mate and stopping when it hits an integer or bracket whose mate is to the left of the parenthesis' mate or when it passes the last integer or bracket. The procedure for passing from one merged word to the next is described in more detail in [29]. This procedure involves deleting a parenthesis when it reaches its rightmost position and then, when a parenthesis has been moved to the right, inserting all the deleted parentheses in their leftmost positions. Since in the worst case all the parentheses may get deleted and reinserted in passing from one word to the next, the algorithm runs in $O(e^2)$ worst-case time if the letters following a deleted parenthesis are pulled to the left as in [29]. Now I replace each deleted parenthesis by a marker (-2). After a parenthesis has been moved to the right, some of the slots between successive undeleted parentheses (or to the left of the leftmost parentheses or to the right of the rightmost one) will contain both markers and either integers or brackets. In each such slot I move all the markers to the left side of the slot and all the integers or brackets to the right side and then replace all the markers by the deleted parentheses, so that the algorithm runs in $O(e)$ worst-case time.

To generate the sensed maps with e edges and v vertices, I generate the rooted maps with e edges and v vertices, or rather, their Lehman code-words, and then I check each one for lexicographical maximality with respect to the code-words for all the rootings of the same map. To this end, I decode the code-word into a rooted map represented by two arrays VERT and NEXT, where the darts are the indices $1, 2, \ldots, 2e$, the ith edge encountered during the decoding procedure consists of the darts $i \leq e$ and $2e+1-i$, VERT[i] is the label assigned to the vertex containing the dart i, NEXT[i] is $P(i)$ and the root is dart 1. Then, I code this map rooted at each of the other darts and compare the new code-word with the original one. Of course, it is not usually necessary to try every dart or even to complete each coding procedure. Since the order is lexicographical, as soon as a letter in the new code-word differs from a letter in the same position in the old one I can terminate the coding; if the new letter is bigger, the old code-word is not maximal and I reject it, and if the new letter is smaller, I try the next dart. If all the darts have been tried and the old code-word hasn't been rejected, I accept (count) it as the representative rooted map of a sensed map.

The decoding and coding procedures each run in $O(e)$ time so that the testing procedure runs in $O(e^2)$ time in the worst case: if the map has $2e$ automorphisms, then all the $2e$ codes for this map are identical, so that all the darts must be tried and each code-word must be constructed in its entirety. But, as we will show, the average time for the testing procedure is $O(e \ln e)$.

Almost all maps have only the trivial automorphism [21]; so in almost all cases the $2e$ words that code the same map rooted at each of the darts will be distinct. If the old code-word is the ith smallest one among those $2e$ words, this process can be modelled by removing balls at random without replacement from an urn containing $i-1$ black balls (words smaller than the old one) and $2e - i$ white ones (words bigger than the old one) until either a white ball is removed or all the balls are removed. If instead the black balls are replaced, the probability that the next ball will be white decreases, so that the expected number of removals increases. The upper bound thus obtained for the expect number of removals is easy to calculate: it is

$$p + 2(1-p)p + 3(1-p)^2 p + \ldots = 1/p, \tag{2}$$

where p is the probability of removing a white ball, which is $(2e-i)/(2e-1)$. If $i = 2e$, (2) is not defined, but in this case (when all the balls are black because the old code-word is maximal) (2) is replaced by the number of removals without replacement, which is $2e$. Since the words coding a given map rooted at all its darts will be generated, i will assume all the values $1, 2, \ldots, 2e$; so the sum of the expected values is less than

$$2e + (2e-1)\,[1/1 + 1/2 + 1/3 + \ldots + 1/(2e-1)]\,, \tag{3}$$

which is asymptotic to $2e \ln e$. The expected number of darts that have to be tried for each generated code word is thus $O(\ln e)$.

To estimate the cost of comparing an old code-word with a new one, let i be the smallest index of a letter in which the new code-word differs from the old one. The expected value of i is given by (2), where p is now the probability that two letters chosen at random from an alphabet are distinct, which is equal to $(a1)/a$, where a is the number of letters in the alphabet. Since the alphabet has at least two letters if $e > 1$, in the average case the number of letters of the new code-word that have to be constructed is bounded by a constant. However, each coding begins by initializing all the vertices to "new", which takes $O(e)$ time; so the average time for testing a code-word for maximality is $O(e \ln e)$. The testing procedure is shown in Figure 1.

To generate the unsensed maps with e edges and v vertices I generate the sensed ones and then, for each sensed map (which I have already constructed by decoding), I reverse the cyclic order of the darts incident to each vertex by constructing the array PREV, where PREV[NEXT[i]] $= i$ for each i. This step runs in $O(e)$ time. Then I code the reversed map at every dart and compare the new code-word with the old maximal code-word (with the same shortcut, and thus the same average-case time-complexity) and accept the old code-word as the representative rooted map of an unsensed map if none of the comparisons have rejected it.

For example, at some point during the generation of the rooted planar maps with 4 edges and 4 vertices the word [( ) (( ))] will be generated. This is the Lehman code-word for the rooted map drawn on the left side of Figure 2, where the darts and the vertices are labelled in the order in which the edges and vertices are encountered during the decoding procedure (dart 1 is the root). The arrays VERT, NEXT and PREV are shown on the right side of Figure 2.

When this map is coded using any of the darts 2, ..., 7 as the root, the first letter is (, which is represented by 0, whereas the first letter of the old code-word is [, represented by

Procedure IsMax (W, a code-word of length $2e$ with p parenthesis pairs)

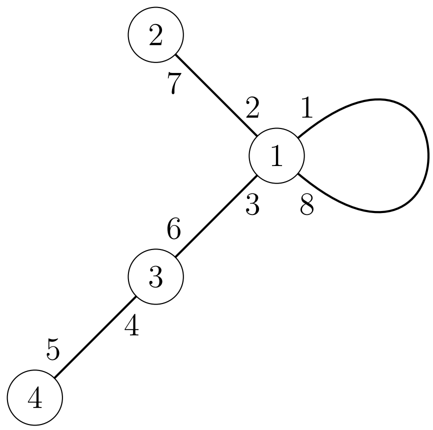
1. Decode W into a map M with e edges and $p+1$ vertices rooted at dart 1
   // $O(e)$ time and space;
2. Calculate the genus g of M; // $O(e)$ time
3. Set Maxword to True;
4. For d from 2 to $2e$ // d is the current dart
5. Initialize the coding of M rooted at d by setting all the vertices to new; // $O(e)$ time
6. For i from 1 to $2e$
7. Set $X[i]$ to the ith letter of the word that codes M rooted at d; // $O(1)$ time
8. If $X[i] > W[i]$ then set Maxword to False and exit loop; // $W < X$; so W is not maximal.
9. If $X[i] < W[i]$ then exit loop; // $X < W$; so this dart need no longer be used as a root.
10. End for i; // $O(e)$ worst case and $O(1)$ average-case time
11. If Maxword = False then exit loop; // W is not maximal; so it will be rejected.
12. End for d; // $O(e)$ worst-case and $O(\ln e)$ average-case number of iterations
    // $O(e^2)$ worst-case and $O(e \ln e)$ average-case time to test W for maximality
13. If Maxword = True then // W is maximal; so it is chosen as the representative of M.
    increase by 1 the number of sensed genus-g maps with e edges and $p+1$ vertices;

End IsMax.

Figure 1: The algorithm for testing whether a code-word represents a sensed map.

2; so the coding terminates immediately. However, when dart 8 is used as the root, the first two letters are [ ]. The second letter of the new code-word is represented by 1, whereas the second letter of the old code-word is represented by 0; so the old code-word is not maximal and is rejected.

Later during the generation of the same set of rooted planar maps the word [ ] ( ) (( )) will be generated. This word codes the same map rooted at dart 8. All of the other darts will yield a lexicographically smaller code; so this word will be accepted as the representative of the map drawn in Figure 2 as a sensed map. But when the cyclic orders are reversed and the dart labelled 1 in the diagram is used as the root, the code-word is [ ] (( )) ( ) . This word first differs from the previous one in the fourth letter, which is represented by 0 in the new word and by 1 in the old word; so the old word will be rejected as an unsensed map as soon as the fourth letter has been computed. But when the new word is generated, it will be accepted as both a sensed map and an unsensed map; so this map will count as two sensed



index i:	1	2	3	4	5	6	7	8
VERT[i]:	1	1	1	3	4	3	2	1
NEXT[i]:	2	3	8	6	5	4	7	1
PREV[i]:	8	1	2	6	5	4	7	3

Figure 2: The planar map rooted at dart 1 coded by [( ) (( ))].

maps and one unsensed map.

4 Generating hypermaps

To generate rooted hypermaps it suffices to generate bicoloured bipartite maps rooted at an edge or, equivalently, rooted at a dart that is incident to a white vertex. This was done by using the following theorem.

Theorem 1. *A rooted map is bipartite if and only if its code-word has the property that between every pair of matching brackets or integers there are an odd number of parentheses.*

Proof. The spanning tree coded by the parenthesis sub-word is bicoloured, with the vertex incident to the root coloured white. A pair of matching brackets or integers is written when the two darts of an edge e that is not in the spanning tree are encountered during the coding process. Each parenthesis between the members of the pair is written when an edge in the spanning tree is traversed, thus passing from a vertex of one colour to a vertex of the other colour. The two darts of e are thus incident to vertices of opposite colours if and only if the number of parentheses between the matching brackets or integers is odd. If this condition holds for every pair of matching brackets or integers, then the map is properly coloured in two colours and is thus bipartite. If this condition is violated for at least one matching pair, then the map is not properly coloured in two colours, and since the colouring of the spanning tree is uniquely determined by the colour of the vertex containing the root, the map cannot be properly coloured in two colours and is thus not bipartite. This completes the proof. □

I modified the program to generate just those code words that both avoid the forbidden sub-word and satisfy the condition stated in the theorem, so that it generates the words coding the rooted bipartite maps that are in bijection with hypermaps of the same genus. To this end, I move brackets or integers from right to left, separating the two members of each pair of brackets or integers by an odd number of parentheses, instead of moving parentheses from left to right as in [29]. To test a code word for maximality, I compare it with all the words coding the same map but with a different root incident to a white vertex, and then with all the words coding the orientation-reversed map with any root incident to a white vertex. In this way I generated all the hypermaps – rooted, sensed, and unsensed – with up to 12 darts.

The time-complexity of the algorithm for generating hypermaps is the same as for maps. Since only the old and the new word have to be stored at any one time and each word is only $O(e)$ letters long, the space-complexity of the generation algorithm is $O(e)$ for both maps and hypermaps. Counting the words and sorting the numbers by genus and the other parameters takes $O(e)$ space for planar maps, $O(e^2)$ space for planar hypermaps and maps that are not necessarily planar, and $O(e^3)$ space for hypermaps that are not necessarily planar.

5 The results of the computation

The work described in [29] was done in 1979 in the Computing Centre of the USSR Academy of Sciences in Moscow on a BESM-6 computer, which has a 10 megahertz clock speed, and users were restricted to 5 minutes of CPU time per run. Within these limitations I was able to do the calculations for maps with up to only 6 edges, processing a total of 110,410 6-edge rooted maps. I published these results, including a table of numbers of sensed and unsensed maps, in [29]. In 2011, using my Macbook Pro laptop, which has a duo processor and a 2.66 gigahertz clock speed, being subject to no run time restrictions, programming in C instead of FORTRAN and optimizing the algorithms, I was able to extend the calculations up to 11 edges; the run time for 11 edges, which processes 285,764,591,114 rooted maps, was about a week. For 10 edges it was about a day, for 9 edges about three hours, for 8 edges about 20 minutes, for 7 edges about 2 minutes, for 6 edges about 10 seconds and for fewer than 6 edges it was too short to be measured. In each case the time was roughly proportional to the number of rooted maps, verifying experimentally the above average-case time complexity for maximality testing. Further verification was provided by the following time trial: it took two minutes to generate all the rooted planar maps with 10 edges and less than six minutes to generate all the unsensed planar maps with 10 edges. For unsensed hypermaps, the computation time was 8 seconds for 9 darts, 2 minutes for 10 darts, 33 minutes for 11 darts and about 10 hours (to process 5,201,061,455 rooted bipartite maps) for 12 darts.

The numbers of rooted maps generated by my program agree with the tables in my joint paper with Prof. Lehman [34]; these tables go up to 11 edges, and the tables in [27] go up to 14 edges. The numbers of sensed maps agree with the numbers calculated jointly with Giorgetti and Mednykh without generating maps; tables for the non-planar maps with up to 11 edges appear in [33] and [31]. The numbers of unsensed maps with up to 6 edges agree with the tables in [29]. The numbers of unsensed planar maps agree with those in unpublished tables given to me by Wormald, who counted those maps and published his results in [36] and [35]. The numbers of rooted and sensed hypermaps of genus 0 and 1 with

d darts agree with those published by Mednykh and Nedela [19]. The numbers of rooted hypermaps of genus 0 and 1, counted by number of vertices, edges and faces, agree with those published by Chauve [7] and Arquès [2], respectively. For unsensed non-planar maps with more than 6 edges, as well as for all the other types of hypermaps, the numbers I generated are, as far as I know, new.

The source code is available as a text file [26]. It will run on any 64-bit computer that runs C programs.

6 Acknowledgment

I wish to thank NSERC for partially supporting this research, and Alain Giorgetti and Alexander Mednykh for suggestions for improving the presentation of this article.

Appendix A: The number of unsensed genus-g maps with e edges and v vertices.

E	v	g=0	g=1	g=2	g=3	g=4	g=5	all g
0	1	1						1
0	sum	1						1
1	1	1						1
1	2	1						1
1	sum	2						2
2	1	1	1					2
2	2	2	0					2
2	3	1	0					1
2	sum	4	1					5
3	1	2	3					5
3	2	5	3					8
3	3	5	0					5
3	4	2	0					2
3	sum	14	6					20
4	1	3	10	4				17
4	2	13	20	0				33
4	3	20	10	0				30
4	4	13	0	0				13
4	5	3	0	0				3
4	sum	52	40	4				96
5	1	6	35	38				79
5	2	35	125	38				198
5	3	83	125	0				208
5	4	83	35	0				118
5	5	35	0	0				35
5	6	6	0	0				6
5	sum	248	320	76				644
6	1	12	132	328	82			554
6	2	104	728	739	0			1571
6	3	340	1226	328	0			1894
6	4	504	728	0	0			1232
6	5	340	132	0	0			472

6	6	104	0	0	0			104
6	7	12	0	0	0			12
6	sum	1416	2946	1395	82			5839
7	1	27	513	2569	2174			5283
7	2	315	4036	9906	2174			16431
7	3	1401	10133	9906	0			21440
7	4	2843	10133	2569	0			15545
7	5	2843	4036	0	0			6879
7	6	1401	513	0	0			1914
7	7	315	0	0	0			315
7	8	27	0	0	0			27
7	sum	9172	29364	24950	4348			67834
8	1	65	2072	18512	37439	7258		65346
8	2	1021	21733	105905	85172	0		213831
8	3	5809	75202	178502	37439	0		296952
8	4	15578	111544	105905	0	0		233027
8	5	21420	75202	18512	0	0		115134
8	6	15578	21733	0	0	0		37311
8	7	5809	2072	0	0	0		7881
8	8	1021	0	0	0	0		1021
8	9	65	0	0	0	0		65
8	sum	66366	309558	427336	160050	7258		970568
9	1	175	8558	124044	488891	344488		966156
9	2	3407	113721	967844	1859361	344488		3288821
9	3	24299	514014	2401662	1859361	0		4799336
9	4	82546	1046261	2401662	488891	0		4019360
9	5	149007	1046261	967844	0	0		2163112
9	6	149007	514014	124044	0	0		787065
9	7	82546	113721	0	0	0		196267
9	8	24299	8558	0	0	0		32857
9	9	3407	0	0	0	0		3407
9	10	175	0	0	0	0		175
9	sum	518868	3365108	6987100	4696504	688976		16256556
10	1	490	35655	781919	5293283	8808724	1491629	16411700
10	2	11814	580810	7887415	29372094	19848849	0	57700982
10	3	102010	3294692	26625471	49022864	8808724	0	87853761
10	4	426879	8728573	39172217	29372094	0	0	77699763
10	5	972660	11966785	26625471	5293283	0	0	44858199
10	6	1273644	8728573	7887415	0	0	0	17889632
10	7	972660	3294692	781919	0	0	0	5049271
10	8	426879	580810	0	0	0	0	1007689
10	9	102010	35655	0	0	0	0	137665
10	10	11814	0	0	0	0	0	11814
10	11	490	0	0	0	0	0	490
10	sum	4301350	37246245	109761827	118353618	37466297	1491629	308620966
11	1	1473	149257	4690016	50026987	159968175	97864389	312700297
11	2	41893	2901436	58891739	374871812	596357213	97864389	1130928482
11	3	429509	20057276	256786053	912749995	596357213	0	1786380046
11	4	2158241	66570286	513820635	912749995	159968175	0	1655267332
11	5	6030752	118697249	513820635	374871812	0	0	1013420448
11	6	9953314	118697249	256786053	50026987	0	0	435463603
11	7	9953314	66570286	58891739	0	0	0	135415339
11	8	6030752	20057276	4690016	0	0	0	30778044
11	9	2158241	2901436	0	0	0	0	5059677
11	10	429509	149257	0	0	0	0	578766
11	11	41893	0	0	0	0	0	41893
11	12	1473	0	0	0	0	0	1473
11	sum	37230364	416751008	1668376886	2675297588	1512650776	195728778	6506035400

Appendix B: The number of hypermaps of genus g.

Number of rooted hypermaps with d darts, v vertices and e edges.

d	v	e	g=0	all g
1	1	1	1	1
1	sum		1	1

Number of sensed hypermaps with d darts, v vertices and e edges.

d	v	e	g=0	all g
1	1	1	1	1
1	sum		1	1

Number of unsensed hypermaps with d darts, v vertices and e edges.

d	v	e	g=0	all g
1	1	1	1	1
1	sum		1	1

Number of rooted hypermaps with d darts, v vertices and e edges.

d	v	e	g=0	all g
2	1	1	1	1
2	1	2	1	1
2	2	1	1	1
2	sum		3	3

Number of sensed hypermaps with d darts, v vertices and e edges.

d	v	e	g=0	all g
2	1	1	1	1
2	1	2	1	1
2	2	1	1	1
2	sum		3	3

Number of unsensed hypermaps with d darts, v vertices and e edges.

d	v	e	g=0	all g
2	1	1	1	1
2	1	2	1	1
2	2	1	1	1
2	sum		3	3

Number of rooted hypermaps with d darts, v vertices and e edges.

d	v	e	g=0	g=1	all g
3	1	1	1	1	2
3	1	2	3	0	3
3	2	1	3	0	3
3	1	3	1	0	1
3	2	2	3	0	3
3	3	1	1	0	1
3	sum		12	1	13

Number of sensed hypermaps with d darts, v vertices and e edges.

d	v	e	g=0	g=1	all g
3	1	1	1	1	2
3	1	2	1	0	1
3	2	1	1	0	1
3	1	3	1	0	1
3	2	2	1	0	1
3	3	1	1	0	1
3	sum		6	1	7

Number of unsensed hypermaps with d darts, v vertices and e edges.

d	v	e	g=0	g=1	all g
3	1	1	1	1	2

3	1	2	1	0	1
3	2	1	1	0	1
3	1	3	1	0	1
3	2	2	1	0	1
3	3	1	1	0	1
3	sum		6	1	7

Number of rooted hypermaps with d darts, v vertices and e edges.

d	v	e	g=0	g=1	all g
4	1	1	1	5	6
4	1	2	6	5	11
4	2	1	6	5	11
4	1	3	6	0	6
4	2	2	17	0	17
4	3	1	6	0	6
4	1	4	1	0	1
4	2	3	6	0	6
4	3	2	6	0	6
4	4	1	1	0	1
4	sum		56	15	71

Number of sensed hypermaps with d darts, v vertices and e edges.

d	v	e	g=0	g=1	all g
4	1	1	1	2	3
4	1	2	2	2	4
4	2	1	2	2	4
4	1	3	2	0	2
4	2	2	5	0	5
4	3	1	2	0	2
4	1	4	1	0	1
4	2	3	2	0	2
4	3	2	2	0	2
4	4	1	1	0	1
4	sum		20	6	26

Number of unsensed hypermaps with d darts, v vertices and e edges.

d	v	e	g=0	g=1	all g
4	1	1	1	2	3
4	1	2	2	2	4
4	2	1	2	2	4
4	1	3	2	0	2
4	2	2	5	0	5
4	3	1	2	0	2
4	1	4	1	0	1
4	2	3	2	0	2
4	3	2	2	0	2
4	4	1	1	0	1
4	sum		20	6	26

Number of rooted hypermaps with d darts, v vertices and e edges.

d	v	e	g=0	g=1	g=2	all g
5	1	1	1	15	8	24
5	1	2	10	40	0	50
5	2	1	10	40	0	50
5	1	3	20	15	0	35
5	2	2	55	40	0	95
5	3	1	20	15	0	35
5	1	4	10	0	0	10
5	2	3	55	0	0	55
5	3	2	55	0	0	55
5	4	1	10	0	0	10
5	1	5	1	0	0	1
5	2	4	10	0	0	10
5	3	3	20	0	0	20
5	4	2	10	0	0	10
5	5	1	1	0	0	1
5	sum		288	165	8	461

Number of sensed hypermaps with d darts, v vertices and e edges.

d	v	e	g=0	g=1	g=2	all g
5	1	1	1	3	4	8
5	1	2	2	8	0	10
5	2	1	2	8	0	10
5	1	3	4	3	0	7
5	2	2	11	8	0	19
5	3	1	4	3	0	7
5	1	4	2	0	0	2
5	2	3	11	0	0	11
5	3	2	11	0	0	11
5	4	1	2	0	0	2
5	1	5	1	0	0	1
5	2	4	2	0	0	2
5	3	3	4	0	0	4
5	4	2	2	0	0	2
5	5	1	1	0	0	1
5	sum		60	33	4	97

Number of unsensed hypermaps with d darts, v vertices and e edges.

d	v	e	g=0	g=1	g=2	all g
5	1	1	1	3	4	8
5	1	2	2	7	0	9
5	2	1	2	7	0	9
5	1	3	4	3	0	7
5	2	2	10	7	0	17
5	3	1	4	3	0	7
5	1	4	2	0	0	2
5	2	3	10	0	0	10

5	3	2	10	0	0	10
5	4	1	2	0	0	2
5	1	5	1	0	0	1
5	2	4	2	0	0	2
5	3	3	4	0	0	4
5	4	2	2	0	0	2
5	5	1	1	0	0	1
5	sum		57	30	4	91

Number of rooted hypermaps with d darts, v vertices and e edges.

d	v	e	g=0	g=1	g=2	all g
6	1	1	1	35	84	120
6	1	2	15	175	84	274
6	2	1	15	175	84	274
6	1	3	50	175	0	225
6	2	2	135	456	0	591
6	3	1	50	175	0	225
6	1	4	50	35	0	85
6	2	3	262	175	0	437
6	3	2	262	175	0	437
6	4	1	50	35	0	85
6	1	5	15	0	0	15
6	2	4	135	0	0	135
6	3	3	262	0	0	262
6	4	2	135	0	0	135
6	5	1	15	0	0	15
6	1	6	1	0	0	1
6	2	5	15	0	0	15
6	3	4	50	0	0	50
6	4	3	50	0	0	50
6	5	2	15	0	0	15
6	6	1	1	0	0	1
6	sum		1584	1611	252	3447

Number of sensed hypermaps with d darts, v vertices and e edges.

d	v	e	g=0	g=1	g=2	all g
6	1	1	1	7	16	24
6	1	2	3	31	16	50
6	2	1	3	31	16	50
6	1	3	10	31	0	41
6	2	2	24	78	0	102
6	3	1	10	31	0	41
6	1	4	10	7	0	17
6	2	3	46	31	0	77
6	3	2	46	31	0	77
6	4	1	10	7	0	17
6	1	5	3	0	0	3
6	2	4	24	0	0	24
6	3	3	46	0	0	46
6	4	2	24	0	0	24
6	5	1	3	0	0	3
6	1	6	1	0	0	1
6	2	5	3	0	0	3
6	3	4	10	0	0	10
6	4	3	10	0	0	10
6	5	2	3	0	0	3
6	6	1	1	0	0	1
6	sum		291	285	48	624

Number of unsensed hypermaps with d darts, v vertices and e edges.

d	v	e	g=0	g=1	g=2	all g
6	1	1	1	6	13	20
6	1	2	3	22	13	38
6	2	1	3	22	13	38
6	1	3	8	22	0	30
6	2	2	21	61	0	82
6	3	1	8	22	0	30
6	1	4	8	6	0	14
6	2	3	36	22	0	58
6	3	2	36	22	0	58
6	4	1	8	6	0	14
6	1	5	3	0	0	3
6	2	4	21	0	0	21
6	3	3	36	0	0	36
6	4	2	21	0	0	21
6	5	1	3	0	0	3
6	1	6	1	0	0	1
6	2	5	3	0	0	3
6	3	4	8	0	0	8
6	4	3	8	0	0	8
6	5	2	3	0	0	3
6	6	1	1	0	0	1
6	sum		240	211	39	490

Number of rooted hypermaps with d darts, v vertices and e edges.

d	v	e	g=0	g=1	g=2	g=3	all g
7	1	1	1	70	469	180	720
7	1	2	21	560	1183	0	1764
7	2	1	21	560	1183	0	1764
7	1	3	105	1050	469	0	1624
7	2	2	280	2695	1183	0	4158
7	3	1	105	1050	469	0	1624
7	1	4	175	560	0	0	735
7	2	3	889	2695	0	0	3584
7	3	2	889	2695	0	0	3584
7	4	1	175	560	0	0	735
7	1	5	105	70	0	0	175
7	2	4	889	560	0	0	1449
7	3	3	1694	1050	0	0	2744
7	4	2	889	560	0	0	1449
7	5	1	105	70	0	0	175
7	1	6	21	0	0	0	21
7	2	5	280	0	0	0	280
7	3	4	889	0	0	0	889
7	4	3	889	0	0	0	889
7	5	2	280	0	0	0	280
7	6	1	21	0	0	0	21
7	1	7	1	0	0	0	1
7	2	6	21	0	0	0	21
7	3	5	105	0	0	0	105
7	4	4	175	0	0	0	175
7	5	3	105	0	0	0	105
7	6	2	21	0	0	0	21
7	7	1	1	0	0	0	1
7	sum		9152	14805	4956	180	29093

Number of sensed hypermaps with d darts, v vertices and e edges.

d	v	e	g=0	g=1	g=2	g=3	all g
7	1	1	1	10	67	30	108
7	1	2	3	80	169	0	252
7	2	1	3	80	169	0	252

7	1	3	15	150	67	0	232
7	2	2	40	385	169	0	594
7	3	1	15	150	67	0	232
7	1	4	25	80	0	0	105
7	2	3	127	385	0	0	512
7	3	2	127	385	0	0	512
7	4	1	25	80	0	0	105
7	1	5	15	10	0	0	25
7	2	4	127	80	0	0	207
7	3	3	242	150	0	0	392
7	4	2	127	80	0	0	207
7	5	1	15	10	0	0	25
7	1	6	3	0	0	0	3
7	2	5	40	0	0	0	40
7	3	4	127	0	0	0	127
7	4	3	127	0	0	0	127
7	5	2	40	0	0	0	40
7	6	1	3	0	0	0	3
7	1	7	1	0	0	0	1
7	2	6	3	0	0	0	3
7	3	5	15	0	0	0	15
7	4	4	25	0	0	0	25
7	5	3	15	0	0	0	15
7	6	2	3	0	0	0	3
7	7	1	1	0	0	0	1
7	sum		1310	2115	708	30	4163

Number of unsensed hypermaps with d darts, v vertices and e edges.

d	v	e	g=0	g=1	g=2	g=3	all g
7	1	1	1	8	44	25	78
7	1	2	3	51	108	0	162
7	2	1	3	51	108	0	162
7	1	3	12	91	44	0	147
7	2	2	33	249	108	0	390
7	3	1	12	91	44	0	147
7	1	4	17	51	0	0	68
7	2	3	90	249	0	0	339
7	3	2	90	249	0	0	339
7	4	1	17	51	0	0	68
7	1	5	12	8	0	0	20
7	2	4	90	51	0	0	141
7	3	3	171	91	0	0	262
7	4	2	90	51	0	0	141
7	5	1	12	8	0	0	20
7	1	6	3	0	0	0	3
7	2	5	33	0	0	0	33
7	3	4	90	0	0	0	90
7	4	3	90	0	0	0	90
7	5	2	33	0	0	0	33
7	6	1	3	0	0	0	3
7	1	7	1	0	0	0	1
7	2	6	3	0	0	0	3
7	3	5	12	0	0	0	12
7	4	4	17	0	0	0	17
7	5	3	12	0	0	0	12
7	6	2	3	0	0	0	3
7	7	1	1	0	0	0	1
7	sum		954	1350	456	25	2785

Number of rooted hypermaps with d darts, v vertices and e edges.

d	v	e	g=0	g=1	g=2	g=3	all g
8	1	1	1	126	1869	3044	5040

8	1	2	28	1470	8526	3044	13068
8	2	1	28	1470	8526	3044	13068
8	1	3	196	4410	8526	0	13132
8	2	2	518	11199	21229	0	32946
8	3	1	196	4410	8526	0	13132
8	1	4	490	4410	1869	0	6769
8	2	3	2436	20684	8526	0	31646
8	3	2	2436	20684	8526	0	31646
8	4	1	490	4410	1869	0	6769
8	1	5	490	1470	0	0	1960
8	2	4	3985	11199	0	0	15184
8	3	3	7500	20684	0	0	28184
8	4	2	3985	11199	0	0	15184
8	5	1	490	1470	0	0	1960
8	1	6	196	126	0	0	322
8	2	5	2436	1470	0	0	3906
8	3	4	7500	4410	0	0	11910
8	4	3	7500	4410	0	0	11910
8	5	2	2436	1470	0	0	3906
8	6	1	196	126	0	0	322
8	1	7	28	0	0	0	28
8	2	6	518	0	0	0	518
8	3	5	2436	0	0	0	2436
8	4	4	3985	0	0	0	3985
8	5	3	2436	0	0	0	2436
8	6	2	518	0	0	0	518
8	7	1	28	0	0	0	28
8	1	8	1	0	0	0	1
8	2	7	28	0	0	0	28
8	3	6	196	0	0	0	196
8	4	5	490	0	0	0	490
8	5	4	490	0	0	0	490
8	6	3	196	0	0	0	196
8	7	2	28	0	0	0	28
8	8	1	1	0	0	0	1
8		sum	54912	131307	77992	9132	273343

Number of sensed hypermaps with d darts, v vertices and e edges.

d	v	e	g=0	g=1	g=2	g=3	all g
8	1	1	1	17	237	385	640
8	1	2	4	187	1072	385	1648
8	2	1	4	187	1072	385	1648
8	1	3	26	557	1072	0	1655
8	2	2	67	1409	2664	0	4140
8	3	1	26	557	1072	0	1655
8	1	4	64	557	237	0	858
8	2	3	309	2597	1072	0	3978
8	3	2	309	2597	1072	0	3978
8	4	1	64	557	237	0	858
8	1	5	64	187	0	0	251
8	2	4	505	1409	0	0	1914
8	3	3	946	2597	0	0	3543
8	4	2	505	1409	0	0	1914
8	5	1	64	187	0	0	251
8	1	6	26	17	0	0	43
8	2	5	309	187	0	0	496
8	3	4	946	557	0	0	1503
8	4	3	946	557	0	0	1503
8	5	2	309	187	0	0	496
8	6	1	26	17	0	0	43
8	1	7	4	0	0	0	4
8	2	6	67	0	0	0	67
8	3	5	309	0	0	0	309

8	4	4	505	0	0	0	505
8	5	3	309	0	0	0	309
8	6	2	67	0	0	0	67
8	7	1	4	0	0	0	4
8	1	8	1	0	0	0	1
8	2	7	4	0	0	0	4
8	3	6	26	0	0	0	26
8	4	5	64	0	0	0	64
8	5	4	64	0	0	0	64
8	6	3	26	0	0	0	26
8	7	2	4	0	0	0	4
8	8	1	1	0	0	0	1
8	sum		6975	16533	9807	1155	34470

Number of unsensed hypermaps with d darts, v vertices and e edges.

d	v	e	g=0	g=1	g=2	g=3	all g
8	1	1	1	13	140	226	380
8	1	2	4	112	596	226	938
8	2	1	4	112	596	226	938
8	1	3	19	314	596	0	929
8	2	2	54	840	1558	0	2452
8	3	1	19	314	596	0	929
8	1	4	41	314	140	0	495
8	2	3	205	1507	596	0	2308
8	3	2	205	1507	596	0	2308
8	4	1	41	314	140	0	495
8	1	5	41	112	0	0	153
8	2	4	325	840	0	0	1165
8	3	3	604	1507	0	0	2111
8	4	2	325	840	0	0	1165
8	5	1	41	112	0	0	153
8	1	6	19	13	0	0	32
8	2	5	205	112	0	0	317
8	3	4	604	314	0	0	918
8	4	3	604	314	0	0	918
8	5	2	205	112	0	0	317
8	6	1	19	13	0	0	32
8	1	7	4	0	0	0	4
8	2	6	54	0	0	0	54
8	3	5	205	0	0	0	205
8	4	4	325	0	0	0	325
8	5	3	205	0	0	0	205
8	6	2	54	0	0	0	54
8	7	1	4	0	0	0	4
8	1	8	1	0	0	0	1
8	2	7	4	0	0	0	4
8	3	6	19	0	0	0	19
8	4	5	41	0	0	0	41
8	5	4	41	0	0	0	41
8	6	3	19	0	0	0	19
8	7	2	4	0	0	0	4
8	8	1	1	0	0	0	1
8	sum		4566	9636	5554	678	20434

Number of rooted hypermaps with d darts, v vertices and e edges.

d	v	e	g=0	g=1	g=2	g=3	g=4	all g
9	1	1	1	210	5985	26060	8064	40320
9	1	2	36	3360	42588	63600	0	109584
9	2	1	36	3360	42588	63600	0	109584
9	1	3	336	14700	77028	26060	0	118124
9	2	2	882	37035	189999	63600	0	291516
9	3	1	336	14700	77028	26060	0	118124

9	1	4	1176	23520	42588	0	0	67284
9	2	3	5754	108285	189999	0	0	304038
9	3	2	5754	108285	189999	0	0	304038
9	4	1	1176	23520	42588	0	0	67284
9	1	5	1764	14700	5985	0	0	22449
9	2	4	13941	108285	42588	0	0	164814
9	3	3	26004	197896	77028	0	0	300928
9	4	2	13941	108285	42588	0	0	164814
9	5	1	1764	14700	5985	0	0	22449
9	1	6	1176	3360	0	0	0	4536
9	2	5	13941	37035	0	0	0	50976
9	3	4	42015	108285	0	0	0	150300
9	4	3	42015	108285	0	0	0	150300
9	5	2	13941	37035	0	0	0	50976
9	6	1	1176	3360	0	0	0	4536
9	1	7	336	210	0	0	0	546
9	2	6	5754	3360	0	0	0	9114
9	3	5	26004	14700	0	0	0	40704
9	4	4	42015	23520	0	0	0	65535
9	5	3	26004	14700	0	0	0	40704
9	6	2	5754	3360	0	0	0	9114
9	7	1	336	210	0	0	0	546
9	1	8	36	0	0	0	0	36
9	2	7	882	0	0	0	0	882
9	3	6	5754	0	0	0	0	5754
9	4	5	13941	0	0	0	0	13941
9	5	4	13941	0	0	0	0	13941
9	6	3	5754	0	0	0	0	5754
9	7	2	882	0	0	0	0	882
9	8	1	36	0	0	0	0	36
9	1	9	1	0	0	0	0	1
9	2	8	36	0	0	0	0	36
9	3	7	336	0	0	0	0	336
9	4	6	1176	0	0	0	0	1176
9	5	5	1764	0	0	0	0	1764
9	6	4	1176	0	0	0	0	1176
9	7	3	336	0	0	0	0	336
9	8	2	36	0	0	0	0	36
9	9	1	1	0	0	0	0	1
9		sum	339456	1138261	1074564	268980	8064	2829325

Number of sensed hypermaps with d darts, v vertices and e edges.

d	v	e	g=0	g=1	g=2	g=3	g=4	all g
9	1	1	1	24	667	2900	900	4492
9	1	2	4	374	4736	7070	0	12184
9	2	1	4	374	4736	7070	0	12184
9	1	3	38	1634	8560	2900	0	13132
9	2	2	98	4115	21113	7070	0	32396
9	3	1	38	1634	8560	2900	0	13132
9	1	4	132	2616	4736	0	0	7484
9	2	3	640	12033	21113	0	0	33786
9	3	2	640	12033	21113	0	0	33786
9	4	1	132	2616	4736	0	0	7484
9	1	5	196	1634	667	0	0	2497
9	2	4	1549	12033	4736	0	0	18318
9	3	3	2890	21990	8560	0	0	33440
9	4	2	1549	12033	4736	0	0	18318
9	5	1	196	1634	667	0	0	2497
9	1	6	132	374	0	0	0	506
9	2	5	1549	4115	0	0	0	5664
9	3	4	4671	12033	0	0	0	16704
9	4	3	4671	12033	0	0	0	16704
9	5	2	1549	4115	0	0	0	5664

9	6	1	132	374	0	0	0	506
9	1	7	38	24	0	0	0	62
9	2	6	640	374	0	0	0	1014
9	3	5	2890	1634	0	0	0	4524
9	4	4	4671	2616	0	0	0	7287
9	5	3	2890	1634	0	0	0	4524
9	6	2	640	374	0	0	0	1014
9	7	1	38	24	0	0	0	62
9	1	8	4	0	0	0	0	4
9	2	7	98	0	0	0	0	98
9	3	6	640	0	0	0	0	640
9	4	5	1549	0	0	0	0	1549
9	5	4	1549	0	0	0	0	1549
9	6	3	640	0	0	0	0	640
9	7	2	98	0	0	0	0	98
9	8	1	4	0	0	0	0	4
9	1	9	1	0	0	0	0	1
9	2	8	4	0	0	0	0	4
9	3	7	38	0	0	0	0	38
9	4	6	132	0	0	0	0	132
9	5	5	196	0	0	0	0	196
9	6	4	132	0	0	0	0	132
9	7	3	38	0	0	0	0	38
9	8	2	4	0	0	0	0	4
9	9	1	1	0	0	0	0	1
9		sum	37746	126501	119436	29910	900	314493

Number of unsensed hypermaps with d darts, v vertices and e edges.

d	v	e	g=0	g=1	g=2	g=3	g=4	all g
9	1	1	1	17	366	1530	524	2438
9	1	2	4	213	2500	3759	0	6476
9	2	1	4	213	2500	3759	0	6476
9	1	3	27	879	4474	1530	0	6910
9	2	2	76	2309	11286	3759	0	17430
9	3	1	27	879	4474	1530	0	6910
9	1	4	78	1388	2500	0	0	3966
9	2	3	403	6568	11286	0	0	18257
9	3	2	403	6568	11286	0	0	18257
9	4	1	78	1388	2500	0	0	3966
9	1	5	116	879	366	0	0	1361
9	2	4	920	6568	2500	0	0	9988
9	3	3	1743	12067	4474	0	0	18284
9	4	2	920	6568	2500	0	0	9988
9	5	1	116	879	366	0	0	1361
9	1	6	78	213	0	0	0	291
9	2	5	920	2309	0	0	0	3229
9	3	4	2747	6568	0	0	0	9315
9	4	3	2747	6568	0	0	0	9315
9	5	2	920	2309	0	0	0	3229
9	6	1	78	213	0	0	0	291
9	1	7	27	17	0	0	0	44
9	2	6	403	213	0	0	0	616
9	3	5	1743	879	0	0	0	2622
9	4	4	2747	1388	0	0	0	4135
9	5	3	1743	879	0	0	0	2622
9	6	2	403	213	0	0	0	616
9	7	1	27	17	0	0	0	44
9	1	8	4	0	0	0	0	4
9	2	7	76	0	0	0	0	76
9	3	6	403	0	0	0	0	403
9	4	5	920	0	0	0	0	920
9	5	4	920	0	0	0	0	920
9	6	3	403	0	0	0	0	403

9	7	2	76	0	0	0	0	76
9	8	1	4	0	0	0	0	4
9	1	9	1	0	0	0	0	1
9	2	8	4	0	0	0	0	4
9	3	7	27	0	0	0	0	27
9	4	6	78	0	0	0	0	78
9	5	5	116	0	0	0	0	116
9	6	4	78	0	0	0	0	78
9	7	3	27	0	0	0	0	27
9	8	2	4	0	0	0	0	4
9	9	1	1	0	0	0	0	1
9		sum	22641	69169	63378	15867	524	171579

Number of rooted hypermaps with d darts, v vertices and e edges.

d	v	e	g=0	g=1	g=2	g=3	g=4	all g
10	1	1	1	330	16401	152900	193248	362880
10	1	2	45	6930	167013	659340	193248	1026576
10	2	1	45	6930	167013	659340	193248	1026576
10	1	3	540	41580	471240	659340	0	1172700
10	2	2	1410	104115	1154095	1595480	0	2855100
10	3	1	540	41580	471240	659340	0	1172700
10	1	4	2520	97020	471240	152900	0	723680
10	2	3	12180	440440	2068070	659340	0	3180030
10	3	2	12180	440440	2068070	659340	0	3180030
10	4	1	2520	97020	471240	152900	0	723680
10	1	5	5292	97020	167013	0	0	269325
10	2	4	40935	697250	1154095	0	0	1892280
10	3	3	75840	1264310	2068070	0	0	3408220
10	4	2	40935	697250	1154095	0	0	1892280
10	5	1	5292	97020	167013	0	0	269325
10	1	6	5292	41580	16401	0	0	63273
10	2	5	60626	440440	167013	0	0	668079
10	3	4	179860	1264310	471240	0	0	1915410
10	4	3	179860	1264310	471240	0	0	1915410
10	5	2	60626	440440	167013	0	0	668079
10	6	1	5292	41580	16401	0	0	63273
10	1	7	2520	6930	0	0	0	9450
10	2	6	40935	104115	0	0	0	145050
10	3	5	179860	440440	0	0	0	620300
10	4	4	288025	697250	0	0	0	985275
10	5	3	179860	440440	0	0	0	620300
10	6	2	40935	104115	0	0	0	145050
10	7	1	2520	6930	0	0	0	9450
10	1	8	540	330	0	0	0	870
10	2	7	12180	6930	0	0	0	19110
10	3	6	75840	41580	0	0	0	117420
10	4	5	179860	97020	0	0	0	276880
10	5	4	179860	97020	0	0	0	276880
10	6	3	75840	41580	0	0	0	117420
10	7	2	12180	6930	0	0	0	19110
10	8	1	540	330	0	0	0	870
10	1	9	45	0	0	0	0	45
10	2	8	1410	0	0	0	0	1410
10	3	7	12180	0	0	0	0	12180
10	4	6	40935	0	0	0	0	40935
10	5	5	60626	0	0	0	0	60626
10	6	4	40935	0	0	0	0	40935
10	7	3	12180	0	0	0	0	12180
10	8	2	1410	0	0	0	0	1410
10	9	1	45	0	0	0	0	45
10	1	10	1	0	0	0	0	1
10	2	9	45	0	0	0	0	45

10	3	8	540	0	0	0	0	540
10	4	7	2520	0	0	0	0	2520
10	5	6	5292	0	0	0	0	5292
10	6	5	5292	0	0	0	0	5292
10	7	4	2520	0	0	0	0	2520
10	8	3	540	0	0	0	0	540
10	9	2	45	0	0	0	0	45
10	10	1	1	0	0	0	0	1
10		sum	2149888	9713835	13545216	6010220	579744	31998903

Number of sensed hypermaps with d darts, v vertices and e edges.

d	v	e	g=0	g=1	g=2	g=3	g=4	all g
10	1	1	1	34	1649	15308	19344	36336
10	1	2	5	698	16725	65972	19344	102744
10	2	1	5	698	16725	65972	19344	102744
10	1	3	56	4172	47164	65972	0	117364
10	2	2	144	10434	115478	159608	0	285664
10	3	1	56	4172	47164	65972	0	117364
10	1	4	256	9724	47164	15308	0	72452
10	2	3	1226	44091	206895	65972	0	318184
10	3	2	1226	44091	206895	65972	0	318184
10	4	1	256	9724	47164	15308	0	72452
10	1	5	536	9724	16725	0	0	26985
10	2	4	4111	69790	115478	0	0	189379
10	3	3	7606	126519	206895	0	0	341020
10	4	2	4111	69790	115478	0	0	189379
10	5	1	536	9724	16725	0	0	26985
10	1	6	536	4172	1649	0	0	6357
10	2	5	6081	44091	16725	0	0	66897
10	3	4	18019	126519	47164	0	0	191702
10	4	3	18019	126519	47164	0	0	191702
10	5	2	6081	44091	16725	0	0	66897
10	6	1	536	4172	1649	0	0	6357
10	1	7	256	698	0	0	0	954
10	2	6	4111	10434	0	0	0	14545
10	3	5	18019	44091	0	0	0	62110
10	4	4	28852	69790	0	0	0	98642
10	5	3	18019	44091	0	0	0	62110
10	6	2	4111	10434	0	0	0	14545
10	7	1	256	698	0	0	0	954
10	1	8	56	34	0	0	0	90
10	2	7	1226	698	0	0	0	1924
10	3	6	7606	4172	0	0	0	11778
10	4	5	18019	9724	0	0	0	27743
10	5	4	18019	9724	0	0	0	27743
10	6	3	7606	4172	0	0	0	11778
10	7	2	1226	698	0	0	0	1924
10	8	1	56	34	0	0	0	90
10	1	9	5	0	0	0	0	5
10	2	8	144	0	0	0	0	144
10	3	7	1226	0	0	0	0	1226
10	4	6	4111	0	0	0	0	4111
10	5	5	6081	0	0	0	0	6081
10	6	4	4111	0	0	0	0	4111
10	7	3	1226	0	0	0	0	1226
10	8	2	144	0	0	0	0	144
10	9	1	5	0	0	0	0	5
10	1	10	1	0	0	0	0	1
10	2	9	5	0	0	0	0	5
10	3	8	56	0	0	0	0	56
10	4	7	256	0	0	0	0	256
10	5	6	536	0	0	0	0	536
10	6	5	536	0	0	0	0	536

10	7	4	256	0	0	0	0	256
10	8	3	56	0	0	0	0	56
10	9	2	5	0	0	0	0	5
10	10	1	1	0	0	0	0	1
10		sum	215602	972441	1355400	601364	58032	3202839

Number of unsensed hypermaps with d darts, v vertices and e edges.

d	v	e	g=0	g=1	g=2	g=3	g=4	all g
10	1	1	1	24	883	7866	9970	18744
10	1	2	5	388	8622	33635	9970	52620
10	2	1	5	388	8622	33635	9970	52620
10	1	3	38	2196	24085	33635	0	59954
10	2	2	110	5676	59772	82472	0	148030
10	3	1	38	2196	24085	33635	0	59954
10	1	4	148	5037	24085	7866	0	37136
10	2	3	746	23303	106787	33635	0	164471
10	3	2	746	23303	106787	33635	0	164471
10	4	1	148	5037	24085	7866	0	37136
10	1	5	298	5037	8622	0	0	13957
10	2	4	2344	36669	59772	0	0	98785
10	3	3	4386	66787	106787	0	0	177960
10	4	2	2344	36669	59772	0	0	98785
10	5	1	298	5037	8622	0	0	13957
10	1	6	298	2196	883	0	0	3377
10	2	5	3391	23303	8622	0	0	35316
10	3	4	10097	66787	24085	0	0	100969
10	4	3	10097	66787	24085	0	0	100969
10	5	2	3391	23303	8622	0	0	35316
10	6	1	298	2196	883	0	0	3377
10	1	7	148	388	0	0	0	536
10	2	6	2344	5676	0	0	0	8020
10	3	5	10097	23303	0	0	0	33400
10	4	4	16103	36669	0	0	0	52772
10	5	3	10097	23303	0	0	0	33400
10	6	2	2344	5676	0	0	0	8020
10	7	1	148	388	0	0	0	536
10	1	8	38	24	0	0	0	62
10	2	7	746	388	0	0	0	1134
10	3	6	4386	2196	0	0	0	6582
10	4	5	10097	5037	0	0	0	15134
10	5	4	10097	5037	0	0	0	15134
10	6	3	4386	2196	0	0	0	6582
10	7	2	746	388	0	0	0	1134
10	8	1	38	24	0	0	0	62
10	1	9	5	0	0	0	0	5
10	2	8	110	0	0	0	0	110
10	3	7	746	0	0	0	0	746
10	4	6	2344	0	0	0	0	2344
10	5	5	3391	0	0	0	0	3391
10	6	4	2344	0	0	0	0	2344
10	7	3	746	0	0	0	0	746
10	8	2	110	0	0	0	0	110
10	9	1	5	0	0	0	0	5
10	1	10	1	0	0	0	0	1
10	2	9	5	0	0	0	0	5
10	3	8	38	0	0	0	0	38
10	4	7	148	0	0	0	0	148
10	5	6	298	0	0	0	0	298
10	6	5	298	0	0	0	0	298
10	7	4	148	0	0	0	0	148
10	8	3	38	0	0	0	0	38
10	9	2	5	0	0	0	0	5
10	10	1	1	0	0	0	0	1

10	sum	121823	513012	698568	307880	29910	1671193	

Number of rooted hypermaps with d darts, v vertices and e edges.

d	v	e	g=0	g=1	g=2	g=3	g=4	g=5	all g
11	1	1	1	495	39963	696905	2286636	604800	3628800
11	1	2	55	13200	550011	4606910	5458464	0	10628640
11	2	1	55	13200	550011	4606910	5458464	0	10628640
11	1	3	825	103950	2221065	8141100	2286636	0	12753576
11	2	2	2145	259017	5409019	19571123	5458464	0	30699768
11	3	1	825	103950	2221065	8141100	2286636	0	12753576
11	1	4	4950	332640	3465000	4606910	0	0	8409500
11	2	3	23694	1493525	15014846	19571123	0	0	36103188
11	3	2	23694	1493525	15014846	19571123	0	0	36103188
11	4	1	4950	332640	3465000	4606910	0	0	8409500
11	1	5	13860	485100	2221065	696905	0	0	3416930
11	2	4	105435	3420835	15014846	4606910	0	0	23148026
11	3	3	194304	6165478	26717482	8141100	0	0	41218364
11	4	2	105435	3420835	15014846	4606910	0	0	23148026
11	5	1	13860	485100	2221065	696905	0	0	3416930
11	1	6	19404	332640	550011	0	0	0	902055
11	2	5	216601	3420835	5409019	0	0	0	9046455
11	3	4	634865	9684433	15014846	0	0	0	25334144
11	4	3	634865	9684433	15014846	0	0	0	25334144
11	5	2	216601	3420835	5409019	0	0	0	9046455
11	6	1	19404	332640	550011	0	0	0	902055
11	1	7	13860	103950	39963	0	0	0	157773
11	2	6	216601	1493525	550011	0	0	0	2260137
11	3	5	931854	6165478	2221065	0	0	0	9318397
11	4	4	1482250	9684433	3465000	0	0	0	14631683
11	5	3	931854	6165478	2221065	0	0	0	9318397
11	6	2	216601	1493525	550011	0	0	0	2260137
11	7	1	13860	103950	39963	0	0	0	157773
11	1	8	4950	13200	0	0	0	0	18150
11	2	7	105435	259017	0	0	0	0	364452
11	3	6	634865	1493525	0	0	0	0	2128390
11	4	5	1482250	3420835	0	0	0	0	4903085
11	5	4	1482250	3420835	0	0	0	0	4903085
11	6	3	634865	1493525	0	0	0	0	2128390
11	7	2	105435	259017	0	0	0	0	364452
11	8	1	4950	13200	0	0	0	0	18150
11	1	9	825	495	0	0	0	0	1320
11	2	8	23694	13200	0	0	0	0	36894
11	3	7	194304	103950	0	0	0	0	298254
11	4	6	634865	332640	0	0	0	0	967505
11	5	5	931854	485100	0	0	0	0	1416954
11	6	4	634865	332640	0	0	0	0	967505
11	7	3	194304	103950	0	0	0	0	298254
11	8	2	23694	13200	0	0	0	0	36894
11	9	1	825	495	0	0	0	0	1320
11	1	10	55	0	0	0	0	0	55
11	2	9	2145	0	0	0	0	0	2145
11	3	8	23694	0	0	0	0	0	23694
11	4	7	105435	0	0	0	0	0	105435
11	5	6	216601	0	0	0	0	0	216601
11	6	5	216601	0	0	0	0	0	216601
11	7	4	105435	0	0	0	0	0	105435
11	8	3	23694	0	0	0	0	0	23694
11	9	2	2145	0	0	0	0	0	2145
11	10	1	55	0	0	0	0	0	55
11	1	11	1	0	0	0	0	0	1
11	2	10	55	0	0	0	0	0	55
11	3	9	825	0	0	0	0	0	825

11	4	8	4950	0	0	0	0	0	4950
11	5	7	13860	0	0	0	0	0	13860
11	6	6	19404	0	0	0	0	0	19404
11	7	5	13860	0	0	0	0	0	13860
11	8	4	4950	0	0	0	0	0	4950
11	9	3	825	0	0	0	0	0	825
11	10	2	55	0	0	0	0	0	55
11	11	1	1	0	0	0	0	0	1
11		sum	13891584	81968469	160174960	112868844	23235300	604800	392743957

Number of sensed hypermaps with d darts, v vertices and e edges.

d	v	e	g=0	g=1	g=2	g=3	g=4	g=5	all g
11	1	1	1	45	3633	63355	207876	54990	329900
11	1	2	5	1200	50001	418810	496224	0	966240
11	2	1	5	1200	50001	418810	496224	0	966240
11	1	3	75	9450	201915	740100	207876	0	1159416
11	2	2	195	23547	491729	1779193	496224	0	2790888
11	3	1	75	9450	201915	740100	207876	0	1159416
11	1	4	450	30240	315000	418810	0	0	764500
11	2	3	2154	135775	1364986	1779193	0	0	3282108
11	3	2	2154	135775	1364986	1779193	0	0	3282108
11	4	1	450	30240	315000	418810	0	0	764500
11	1	5	1260	44100	201915	63355	0	0	310630
11	2	4	9585	310985	1364986	418810	0	0	2104366
11	3	3	17664	560498	2428862	740100	0	0	3747124
11	4	2	9585	310985	1364986	418810	0	0	2104366
11	5	1	1260	44100	201915	63355	0	0	310630
11	1	6	1764	30240	50001	0	0	0	82005
11	2	5	19691	310985	491729	0	0	0	822405
11	3	4	57715	880403	1364986	0	0	0	2303104
11	4	3	57715	880403	1364986	0	0	0	2303104
11	5	2	19691	310985	491729	0	0	0	822405
11	6	1	1764	30240	50001	0	0	0	82005
11	1	7	1260	9450	3633	0	0	0	14343
11	2	6	19691	135775	50001	0	0	0	205467
11	3	5	84714	560498	201915	0	0	0	847127
11	4	4	134750	880403	315000	0	0	0	1330153
11	5	3	84714	560498	201915	0	0	0	847127
11	6	2	19691	135775	50001	0	0	0	205467
11	7	1	1260	9450	3633	0	0	0	14343
11	1	8	450	1200	0	0	0	0	1650
11	2	7	9585	23547	0	0	0	0	33132
11	3	6	57715	135775	0	0	0	0	193490
11	4	5	134750	310985	0	0	0	0	445735
11	5	4	134750	310985	0	0	0	0	445735
11	6	3	57715	135775	0	0	0	0	193490
11	7	2	9585	23547	0	0	0	0	33132
11	8	1	450	1200	0	0	0	0	1650
11	1	9	75	45	0	0	0	0	120
11	2	8	2154	1200	0	0	0	0	3354
11	3	7	17664	9450	0	0	0	0	27114
11	4	6	57715	30240	0	0	0	0	87955
11	5	5	84714	44100	0	0	0	0	128814
11	6	4	57715	30240	0	0	0	0	87955
11	7	3	17664	9450	0	0	0	0	27114
11	8	2	2154	1200	0	0	0	0	3354
11	9	1	75	45	0	0	0	0	120
11	1	10	5	0	0	0	0	0	5
11	2	9	195	0	0	0	0	0	195
11	3	8	2154	0	0	0	0	0	2154
11	4	7	9585	0	0	0	0	0	9585
11	5	6	19691	0	0	0	0	0	19691
11	6	5	19691	0	0	0	0	0	19691

11	7	4	9585	0	0	0	0	0	9585
11	8	3	2154	0	0	0	0	0	2154
11	9	2	195	0	0	0	0	0	195
11	10	1	5	0	0	0	0	0	5
11	1	11	1	0	0	0	0	0	1
11	2	10	5	0	0	0	0	0	5
11	3	9	75	0	0	0	0	0	75
11	4	8	450	0	0	0	0	0	450
11	5	7	1260	0	0	0	0	0	1260
11	6	6	1764	0	0	0	0	0	1764
11	7	5	1260	0	0	0	0	0	1260
11	8	4	450	0	0	0	0	0	450
11	9	3	75	0	0	0	0	0	75
11	10	2	5	0	0	0	0	0	5
11	11	1	1	0	0	0	0	0	1
11		sum	1262874	7451679	14561360	10260804	2112300	54990	35704007

Number of unsensed hypermaps with d darts, v vertices and e edges.

d	v	e	g=0	g=1	g=2	g=3	g=4	g=5	all g
11	1	1	1	30	1894	32028	104748	28169	166870
11	1	2	5	650	25442	211149	250674	0	487920
11	2	1	5	650	25442	211149	250674	0	487920
11	1	3	50	4890	102033	372579	104748	0	584300
11	2	2	145	12507	250375	899919	250674	0	1413620
11	3	1	50	4890	102033	372579	104748	0	584300
11	1	4	250	15429	158902	211149	0	0	385730
11	2	3	1272	70364	692895	899919	0	0	1664450
11	3	2	1272	70364	692895	899919	0	0	1664450
11	4	1	250	15429	158902	211149	0	0	385730
11	1	5	680	22439	102033	32028	0	0	157180
11	2	4	5280	159881	692895	211149	0	0	1069205
11	3	3	9895	289690	1235766	372579	0	0	1907930
11	4	2	5280	159881	692895	211149	0	0	1069205
11	5	1	680	22439	102033	32028	0	0	157180
11	1	6	932	15429	25442	0	0	0	41803
11	2	5	10580	159881	250375	0	0	0	420836
11	3	4	31276	453914	692895	0	0	0	1178085
11	4	3	31276	453914	692895	0	0	0	1178085
11	5	2	10580	159881	250375	0	0	0	420836
11	6	1	932	15429	25442	0	0	0	41803
11	1	7	680	4890	1894	0	0	0	7464
11	2	6	10580	70364	25442	0	0	0	106386
11	3	5	45593	289690	102033	0	0	0	437316
11	4	4	72417	453914	158902	0	0	0	685233
11	5	3	45593	289690	102033	0	0	0	437316
11	6	2	10580	70364	25442	0	0	0	106386
11	7	1	680	4890	1894	0	0	0	7464
11	1	8	250	650	0	0	0	0	900
11	2	7	5280	12507	0	0	0	0	17787
11	3	6	31276	70364	0	0	0	0	101640
11	4	5	72417	159881	0	0	0	0	232298
11	5	4	72417	159881	0	0	0	0	232298
11	6	3	31276	70364	0	0	0	0	101640
11	7	2	5280	12507	0	0	0	0	17787
11	8	1	250	650	0	0	0	0	900
11	1	9	50	30	0	0	0	0	80
11	2	8	1272	650	0	0	0	0	1922
11	3	7	9895	4890	0	0	0	0	14785
11	4	6	31276	15429	0	0	0	0	46705
11	5	5	45593	22439	0	0	0	0	68032
11	6	4	31276	15429	0	0	0	0	46705
11	7	3	9895	4890	0	0	0	0	14785
11	8	2	1272	650	0	0	0	0	1922

11	9	1	50	30	0	0	0	0	80
11	1	10	5	0	0	0	0	0	5
11	2	9	145	0	0	0	0	0	145
11	3	8	1272	0	0	0	0	0	1272
11	4	7	5280	0	0	0	0	0	5280
11	5	6	10580	0	0	0	0	0	10580
11	6	5	10580	0	0	0	0	0	10580
11	7	4	5280	0	0	0	0	0	5280
11	8	3	1272	0	0	0	0	0	1272
11	9	2	145	0	0	0	0	0	145
11	10	1	5	0	0	0	0	0	5
11	1	11	1	0	0	0	0	0	1
11	2	10	5	0	0	0	0	0	5
11	3	9	50	0	0	0	0	0	50
11	4	8	250	0	0	0	0	0	250
11	5	7	680	0	0	0	0	0	680
11	6	6	932	0	0	0	0	0	932
11	7	5	680	0	0	0	0	0	680
11	8	4	250	0	0	0	0	0	250
11	9	3	50	0	0	0	0	0	50
11	10	2	5	0	0	0	0	0	5
11	11	1	1	0	0	0	0	0	1
11		sum	683307	3843024	7391499	5180472	1066266	28169	18192737

Number of rooted hypermaps with d darts, v vertices and e edges.

d	v	e	g=0	g=1	g=2	g=3	g=4	g=5	all g
12	1	1	1	715	88803	2641925	18128396	19056960	39916800
12	1	2	66	23595	1585584	24656775	75220860	19056960	120543840
12	2	1	66	23595	1585584	24656775	75220860	19056960	120543840
12	1	3	1210	235950	8654646	66805310	75220860	0	150917976
12	2	2	3135	585585	20981337	159762815	178462816	0	359795688
12	3	1	1210	235950	8654646	66805310	75220860	0	150917976
12	1	4	9075	990990	19324305	66805310	18128396	0	105258076
12	2	3	43098	4410120	82897296	280514670	75220860	0	443086044
12	3	2	43098	4410120	82897296	280514670	75220860	0	443086044
12	4	1	9075	990990	19324305	66805310	18128396	0	105258076
12	1	5	32670	1981980	19324305	24656775	0	0	45995730
12	2	4	245223	13768300	128420004	159762815	0	0	302196342
12	3	3	449988	24695580	227256510	280514670	0	0	532916748
12	4	2	245223	13768300	128420004	159762815	0	0	302196342
12	5	1	32670	1981980	19324305	24656775	0	0	45995730
12	1	6	60984	1981980	8654646	2641925	0	0	13339535
12	2	5	666996	19920390	82897296	24656775	0	0	128141457
12	3	4	1936308	55785870	227256510	66805310	0	0	351783998
12	4	3	1936308	55785870	227256510	66805310	0	0	351783998
12	5	2	666996	19920390	82897296	24656775	0	0	128141457
12	6	1	60984	1981980	8654646	2641925	0	0	13339535
12	1	7	60984	990990	1585584	0	0	0	2637558
12	2	6	925190	13768300	20981337	0	0	0	35674827
12	3	5	3915576	55785870	82897296	0	0	0	142598742
12	4	4	6195560	87100531	128420004	0	0	0	221716095
12	5	3	3915576	55785870	82897296	0	0	0	142598742
12	6	2	925190	13768300	20981337	0	0	0	35674827
12	7	1	60984	990990	1585584	0	0	0	2637558
12	1	8	32670	235950	88803	0	0	0	357423
12	2	7	666996	4410120	1585584	0	0	0	6662700
12	3	6	3915576	24695580	8654646	0	0	0	37265802
12	4	5	9032898	55785870	19324305	0	0	0	84143073
12	5	4	9032898	55785870	19324305	0	0	0	84143073
12	6	3	3915576	24695580	8654646	0	0	0	37265802
12	7	2	666996	4410120	1585584	0	0	0	6662700
12	8	1	32670	235950	88803	0	0	0	357423

12	1	9	9075	23595	0	0	0	0	32670
12	2	8	245223	585585	0	0	0	0	830808
12	3	7	1936308	4410120	0	0	0	0	6346428
12	4	6	6195560	13768300	0	0	0	0	19963860
12	5	5	9032898	19920390	0	0	0	0	28953288
12	6	4	6195560	13768300	0	0	0	0	19963860
12	7	3	1936308	4410120	0	0	0	0	6346428
12	8	2	245223	585585	0	0	0	0	830808
12	9	1	9075	23595	0	0	0	0	32670
12	1	10	1210	715	0	0	0	0	1925
12	2	9	43098	23595	0	0	0	0	66693
12	3	8	449988	235950	0	0	0	0	685938
12	4	7	1936308	990990	0	0	0	0	2927298
12	5	6	3915576	1981980	0	0	0	0	5897556
12	6	5	3915576	1981980	0	0	0	0	5897556
12	7	4	1936308	990990	0	0	0	0	2927298
12	8	3	449988	235950	0	0	0	0	685938
12	9	2	43098	23595	0	0	0	0	66693
12	10	1	1210	715	0	0	0	0	1925
12	1	11	66	0	0	0	0	0	66
12	2	10	3135	0	0	0	0	0	3135
12	3	9	43098	0	0	0	0	0	43098
12	4	8	245223	0	0	0	0	0	245223
12	5	7	666996	0	0	0	0	0	666996
12	6	6	925190	0	0	0	0	0	925190
12	7	5	666996	0	0	0	0	0	666996
12	8	4	245223	0	0	0	0	0	245223
12	9	3	43098	0	0	0	0	0	43098
12	10	2	3135	0	0	0	0	0	3135
12	11	1	66	0	0	0	0	0	66
12	1	12	1	0	0	0	0	0	1
12	2	11	66	0	0	0	0	0	66
12	3	10	1210	0	0	0	0	0	1210
12	4	9	9075	0	0	0	0	0	9075
12	5	8	32670	0	0	0	0	0	32670
12	6	7	60984	0	0	0	0	0	60984
12	7	6	60984	0	0	0	0	0	60984
12	8	5	32670	0	0	0	0	0	32670
12	9	4	9075	0	0	0	0	0	9075
12	10	3	1210	0	0	0	0	0	1210
12	11	2	66	0	0	0	0	0	66
12	12	1	1	0	0	0	0	0	1
12		sum	91287552	685888171	1805010948	1877530740	684173164	57170880	5201061455

Number of sensed hypermaps with d darts, v vertices and e edges.

d	v	e	g=0	g=1	g=2	g=3	g=4	g=5	all g
12	1	1	1	62	7417	220244	1510846	1588218	3326788
12	1	2	6	1976	132202	2054974	6268712	1588218	10046088
12	2	1	6	1976	132202	2054974	6268712	1588218	10046088
12	1	3	104	19694	721382	5567550	6268712	0	12577442
12	2	2	265	48846	1748723	13314231	14872428	0	29984493
12	3	1	104	19694	721382	5567550	6268712	0	12577442
12	1	4	765	82652	1610617	5567550	1510846	0	8772430
12	2	3	3605	367645	6908644	23377106	6268712	0	36925712
12	3	2	3605	367645	6908644	23377106	6268712	0	36925712
12	4	1	765	82652	1610617	5567550	1510846	0	8772430
12	1	5	2736	165262	1610617	2054974	0	0	3833589
12	2	4	20472	1147628	10702449	13314231	0	0	25184780
12	3	3	37545	2058329	18938994	23377106	0	0	44411974
12	4	2	20472	1147628	10702449	13314231	0	0	25184780
12	5	1	2736	165262	1610617	2054974	0	0	3833589
12	1	6	5102	165262	721382	220244	0	0	1111990
12	2	5	55633	1660331	6908644	2054974	0	0	10679582

12	3	4	161455	4649379	18938994	5567550	0	0	29317378
12	4	3	161455	4649379	18938994	5567550	0	0	29317378
12	5	2	55633	1660331	6908644	2054974	0	0	10679582
12	6	1	5102	165262	721382	220244	0	0	1111990
12	1	7	5102	82652	132202	0	0	0	219956
12	2	6	77174	1147628	1748723	0	0	0	2973525
12	3	5	326432	4649379	6908644	0	0	0	11884455
12	4	4	516507	7259140	10702449	0	0	0	18478096
12	5	3	326432	4649379	6908644	0	0	0	11884455
12	6	2	77174	1147628	1748723	0	0	0	2973525
12	7	1	5102	82652	132202	0	0	0	219956
12	1	8	2736	19694	7417	0	0	0	29847
12	2	7	55633	367645	132202	0	0	0	555480
12	3	6	326432	2058329	721382	0	0	0	3106143
12	4	5	752940	4649379	1610617	0	0	0	7012936
12	5	4	752940	4649379	1610617	0	0	0	7012936
12	6	3	326432	2058329	721382	0	0	0	3106143
12	7	2	55633	367645	132202	0	0	0	555480
12	8	1	2736	19694	7417	0	0	0	29847
12	1	9	765	1976	0	0	0	0	2741
12	2	8	20472	48846	0	0	0	0	69318
12	3	7	161455	367645	0	0	0	0	529100
12	4	6	516507	1147628	0	0	0	0	1664135
12	5	5	752940	1660331	0	0	0	0	2413271
12	6	4	516507	1147628	0	0	0	0	1664135
12	7	3	161455	367645	0	0	0	0	529100
12	8	2	20472	48846	0	0	0	0	69318
12	9	1	765	1976	0	0	0	0	2741
12	1	10	104	62	0	0	0	0	166
12	2	9	3605	1976	0	0	0	0	5581
12	3	8	37545	19694	0	0	0	0	57239
12	4	7	161455	82652	0	0	0	0	244107
12	5	6	326432	165262	0	0	0	0	491694
12	6	5	326432	165262	0	0	0	0	491694
12	7	4	161455	82652	0	0	0	0	244107
12	8	3	37545	19694	0	0	0	0	57239
12	9	2	3605	1976	0	0	0	0	5581
12	10	1	104	62	0	0	0	0	166
12	1	11	6	0	0	0	0	0	6
12	2	10	265	0	0	0	0	0	265
12	3	9	3605	0	0	0	0	0	3605
12	4	8	20472	0	0	0	0	0	20472
12	5	7	55633	0	0	0	0	0	55633
12	6	6	77174	0	0	0	0	0	77174
12	7	5	55633	0	0	0	0	0	55633
12	8	4	20472	0	0	0	0	0	20472
12	9	3	3605	0	0	0	0	0	3605
12	10	2	265	0	0	0	0	0	265
12	11	1	6	0	0	0	0	0	6
12	1	12	1	0	0	0	0	0	1
12	2	11	6	0	0	0	0	0	6
12	3	10	104	0	0	0	0	0	104
12	4	9	765	0	0	0	0	0	765
12	5	8	2736	0	0	0	0	0	2736
12	6	7	5102	0	0	0	0	0	5102
12	7	6	5102	0	0	0	0	0	5102
12	8	5	2736	0	0	0	0	0	2736
12	9	4	765	0	0	0	0	0	765
12	10	3	104	0	0	0	0	0	104
12	11	2	6	0	0	0	0	0	6
12	12	1	1	0	0	0	0	0	1
12		sum	7611156	57167260	150429819	156469887	57017238	4764654	433460014

Number of unsensed hypermaps with d darts, v vertices and e edges.

d	v	e	g=0	g=1	g=2	g=3	g=4	g=5	all g
12	1	1	1	41	3836	110914	757977	797345	1670114
12	1	2	6	1058	66865	1031387	3142703	797345	5039364
12	2	1	6	1058	66865	1031387	3142703	797345	5039364
12	1	3	67	10107	362868	2791448	3142703	0	6307193
12	2	2	195	25594	883711	6689591	7472556	0	15071647
12	3	1	67	10107	362868	2791448	3142703	0	6307193
12	1	4	420	41890	808812	2791448	757977	0	4400547
12	2	3	2086	188410	3481842	11744994	3142703	0	18560035
12	3	2	2086	188410	3481842	11744994	3142703	0	18560035
12	4	1	420	41890	808812	2791448	757977	0	4400547
12	1	5	1443	83460	808812	1031387	0	0	1925102
12	2	4	11060	583755	5389906	6689591	0	0	12674312
12	3	3	20565	1050920	9551009	11744994	0	0	22367488
12	4	2	11060	583755	5389906	6689591	0	0	12674312
12	5	1	1443	83460	808812	1031387	0	0	1925102
12	1	6	2651	83460	362868	110914	0	0	559893
12	2	5	29237	842635	3481842	1031387	0	0	5385101
12	3	4	85673	2366909	9551009	2791448	0	0	14795039
12	4	3	85673	2366909	9551009	2791448	0	0	14795039
12	5	2	29237	842635	3481842	1031387	0	0	5385101
12	6	1	2651	83460	362868	110914	0	0	559893
12	1	7	2651	41890	66865	0	0	0	111406
12	2	6	40348	583755	883711	0	0	0	1507814
12	3	5	171275	2366909	3481842	0	0	0	6020026
12	4	4	271482	3696390	5389906	0	0	0	9357778
12	5	3	171275	2366909	3481842	0	0	0	6020026
12	6	2	40348	583755	883711	0	0	0	1507814
12	7	1	2651	41890	66865	0	0	0	111406
12	1	8	1443	10107	3836	0	0	0	15386
12	2	7	29237	188410	66865	0	0	0	284512
12	3	6	171275	1050920	362868	0	0	0	1585063
12	4	5	394258	2366909	808812	0	0	0	3569979
12	5	4	394258	2366909	808812	0	0	0	3569979
12	6	3	171275	1050920	362868	0	0	0	1585063
12	7	2	29237	188410	66865	0	0	0	284512
12	8	1	1443	10107	3836	0	0	0	15386
12	1	9	420	1058	0	0	0	0	1478
12	2	8	11060	25594	0	0	0	0	36654
12	3	7	85673	188410	0	0	0	0	274083
12	4	6	271482	583755	0	0	0	0	855237
12	5	5	394258	842635	0	0	0	0	1236893
12	6	4	271482	583755	0	0	0	0	855237
12	7	3	85673	188410	0	0	0	0	274083
12	8	2	11060	25594	0	0	0	0	36654
12	9	1	420	1058	0	0	0	0	1478
12	1	10	67	41	0	0	0	0	108
12	2	9	2086	1058	0	0	0	0	3144
12	3	8	20565	10107	0	0	0	0	30672
12	4	7	85673	41890	0	0	0	0	127563
12	5	6	171275	83460	0	0	0	0	254735
12	6	5	171275	83460	0	0	0	0	254735
12	7	4	85673	41890	0	0	0	0	127563
12	8	3	20565	10107	0	0	0	0	30672
12	9	2	2086	1058	0	0	0	0	3144
12	10	1	67	41	0	0	0	0	108
12	1	11	6	0	0	0	0	0	6
12	2	10	195	0	0	0	0	0	195
12	3	9	2086	0	0	0	0	0	2086
12	4	8	11060	0	0	0	0	0	11060
12	5	7	29237	0	0	0	0	0	29237
12	6	6	40348	0	0	0	0	0	40348

12	7	5	29237	0	0	0	0	0	29237
12	8	4	11060	0	0	0	0	0	11060
12	9	3	2086	0	0	0	0	0	2086
12	10	2	195	0	0	0	0	0	195
12	11	1	6	0	0	0	0	0	6
12	1	12	1	0	0	0	0	0	1
12	2	11	6	0	0	0	0	0	6
12	3	10	67	0	0	0	0	0	67
12	4	9	420	0	0	0	0	0	420
12	5	8	1443	0	0	0	0	0	1443
12	6	7	2651	0	0	0	0	0	2651
12	7	6	2651	0	0	0	0	0	2651
12	8	5	1443	0	0	0	0	0	1443
12	9	4	420	0	0	0	0	0	420
12	10	3	67	0	0	0	0	0	67
12	11	2	6	0	0	0	0	0	6
12	12	1	1	0	0	0	0	0	1
12		sum	4004055	29107494	75807708	78573507	28602705	2392035	218487504

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2010 *Mathematics Subject Classification*: Primary 05C30; Secondary 05C10, 05C65, 05C85.
Keywords: map, hypermap, exhaustive generation, isomorphism class.

(Concerned with sequences A006385, A006387, A214814, A214815, A214816, A000257, A118093, A214817, A214818, A003319, A090371, A118094, A214819, A214820, A057005, A214821, A214822, A214823, A215017, and A215018.)

Received October 3 2014; revised versions received October 6 2014; October 29 2014; February 19 2015; February 23 2015. Published in *Journal of Integer Sequences*, May 12 2015.

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