## *Research Article*
# Asteroids: Assessing Catastrophic Risks

## Graciela Chichilnisky and Peter Eisenberger

*Columbia University, New York, NY 10027, USA*

Correspondence should be addressed to Graciela Chichilnisky, chichilnisky1@gmail.com

We evaluate two risk profiles: (i) global warming risks and (ii) collisions with asteroids that can cause the extinction of our species. The expected values computed for these two risks suggest that no action will be taken to avoid extinction. The result is somewhat counterintuitive, but it is typical of the results of using classic decision theory to evaluate catastrophic risks in the distant future, see the study by Posner (2004). We establish why expected value is insensitive to catastrophic risks see the study by Chichilnisky (1996), and use another criterion to evaluate risk based on axioms for choice under uncertainty that update the classic Von Neumann theory and require equal treatment for rare and frequent events. Optimizing according to the new criterion is shown to be equivalent to optimizing expected utility with a restriction on the worst outcome in the case of a catastrophe. The evaluation obtained from the new criterion seems more intuitively plausible, and suggests a more practical and realistic approach to catastrophic risks: optimizing expected value while minimizing losses in the case of a catastrophe.

## 1. Asteroids

Sixty five million years ago, an asteroid crashed into earth. Global winds distributed the dust throughout the atmosphere, blocking sunlight, and many life forms that relied on the sun eventually perished. In a short period of time, experts believe, the mighty dinosaurs that dominated our planet went extinct. Realistically the same fate awaits us. Over 99.99% of the species that have ever existed are now extinct [1, 2]. If our species survives long enough, we will be exposed to an asteroid and could suffer the same fate as the dinosaurs. The data suggests that asteroids of that caliber will hit our planet on average once every 100 million years [2]. The last one was 65 million years ago. Under current conditions, when the next one hits the earth, humans and many other species could go extinct.

What should we do about this threat to our survival and others like it? And if the issue is serious, why is this issue getting so little attention whereas the less catastrophic threat of global warming is in the news almost daily?

The purpose of this paper is to provide answers to these questions. We examine systematically how to deal with *catastrophic risks* such as asteroid impacts, which are small-probability events with enormous consequences, events that could threaten the survival of our species, and compare their treatment with risks like global warming that are more imminent and familiar but possibly less catastrophic.

The task is not easy. Classic tools for risk management are notoriously poor for managing catastrophic risks, (see Posner [2] and Chichilnisky [3, 4]). There is an understandable tendency to ignore rare events, such as an asteroid impact, which are unlikely to occur in our lifetimes or those of our families [2, 5]. Yes this is a questionable instinct at this stage of human evolution where our knowledge enables to identify such risks. Standard decision tools make this task difficult. We show using the existing data that a major disturbance caused by global warming of less than 1% of GDP overwhelms in expected value the costs associated with an asteroid impact that can plausibly lead to the extinction of the human species. We show that the expected value of the loss caused by an asteroid that leads to extinction—is between $500 million and $92 billion. A loss of this magnitude is smaller than that of a failure of a single atomic plant—the Russians lost more than $140 billion with the accident at Chernobyl—or with the potential risks involved in global warming that is between $890 billion and $9.7 trillion [2]. Using expected values therefore we are led to believe that preventing asteroid impacts should not rank high in our policy priorities. Common sense rebels against the computation we just provided. The ability to anticipate and plan for threats that have never been experienced by any current or past member of the species and are unlikely to happen in our lifespans, appears to be unique to our species. We need to use a risk management approach that enables us to deal more effectively with such threats [2]. To overcome this problem this paper summarizes a new axiomatic approach to catastrophic risks that updates current methods developed initially by John Von Neumann, see Chichilnisky [3, 4, 6–9], and offers practical figures to evaluate possible policies that would protect us from asteroid impacts. Our conclusion is that we are underinvesting in preventing the risk of asteroid like threats. Much can and should be done at a relatively small cost; this paper suggests a methodology and a range of dollar values that should be spent to protect against such risks to help prevent the extinction of our species.

## 2. Catastrophes and the Survival of the Species

A catastrophe is a rare event with enormous consequences. In a recent book, Posner [2] classifies catastrophes into various types, each of which threats the survival of our species. He uses a classic approach to value the importance of a risk by quantifying its *expected value*, namely, the product of the probability times the loss. For example, the expected value of an event that occurs with ten percent probability and involves $1 billion loss is $10^9 \times 10^{-1} = \$100$ million. This approach is used by actuaries to price the cost of life insurance policies, and is also by law the measure used in US Congress when evaluating budget plans with uncertain outcomes. The notion of *expected value* started with Von Neumann and Morgenstern about 60 years ago [10], and it is based on their axioms or principles for decision making under uncertainty formalized in [11, 12]. Posner [2] uses the concept of *expected value* to evaluate risks but warns the reader about its weaknesses for evaluating catastrophic risks (see Posner [2, Chapter 3, pages 150–154]). This weakness is exposed in the case of asteroids, when we ask how much we should invest in preventing the impact of an asteroid that can destroy all of the earth's economic value forever. Posner [2] argues that *expected value* does not capture

the true impact of such a catastrophe; that something else is at stake. Because of his loyalty to the concept of expected value, which does not work well in these cases, Posner appears to be arguing that rationality does not work in the case of catastrophes, that we cannot deal rationally with small probabilities events that cause such large and irreversible damage.

Perhaps the problem is not one of rationality. There may be a *different* rationality needed when considering the long-range future of the species. It could be that *expected value* is a good measure for evaluating risks that have a good chance to occur in our lifetime, but not for evaluating risks that are important but have essentially a zero chance to occur while we are alive. For such risks we may need another approach overall, for both the present and the future. In our current state of evolution it would seem useful to oppose a human tendency based on our hunter-gatherer origins to give preference to immediate outcomes as opposed to more distant ones; see the study by McClure et al. [5]. When using *expected value* the response we obtain seems to clash with our intuition because the probabilities involved are so small that they render the computation almost meaningless, as seen numerically in the examples provided below. The experimental evidence summarized below provides further support for this view.

## 3. Experimental Evidence and Alternative Approaches

*Expected utility* optimization derives from Von Neumann's (NM) axioms, but it is well known for sometime that it conflicts with the experimental evidence on how humans choose under uncertainty; for example, see the studies by Allais [13], Machina, [14, 15], Tversky and Wakker [16]. Problems arise when there are infrequent events involved; examples are weather catastrophes like hurricanes or mass extinctions. Similar types of conflicts appear when using the standard criterion of *present value optimization* for choosing among projects that evolve through time. Other problems arise when the plans involve very long time horizons as in the disposal of nuclear waste (Heal [17]). While the problem areas mentioned above are quite different, they all share a common mathematical root: the relative insensitivity of the classic axioms of choice towards (1) small-probability events (2) the far away future [3, 4, 6, 17]. The mathematical structure of the problem is the same in all cases: it arises from the use of "normal" distributions; the "bell curves" to describe the frequency with which we expect everything to occur from weather events to returns on investments or corporate profits. Normal distributions arise when many independent events contribute to some outcome. However when there are unexpected interconnections or catastrophic events, normal distribution can understate (1) the role of small-probability events (2) the role of events that are very far into the future. We formalize this problem below and provide a solution.

Taking a leaf from Von Neumann and Morgenstern's book, Chichilnisky [3, 4, 6] reconsidered the foundations of the expected value approach, which are the VNM axioms for choice under uncertainty, see also Arrow [11] and Hernstein and Milnor [12]. A first step is to show that classic axioms can be "biased" against small-probability events, as was established by Chichilnisky in [6]. She introduced new axioms for choice under uncertainty that require more symmetry in the treatment of small and large probability events [3, 4]. The new axioms coincide with those of Von Neumann and Morgenstern when the events involved have "normal" probabilities, for example, when they are likely to occur in our lifetime. But the new axioms give rise to a rather different decision making criterion when the probabilities involved are extremely small, or equivalently when the events are only likely to occur in

a very long future. The two sets of axioms are consistent with each other for "normal" events while they are quite different on catastrophic events. How can this be?

A somewhat far-fetched analogy is the relationship between classical mechanics and general relativity. The former applies to "normal scales" that are closer to own reality on earth, while the latter applies to large-scale phenomena involving astral bodies. Both are correct in their respective spheres, and neither contradicts the other. The same could be the case with the Von Neumann-Morgenstern and the Chichilnisky's axioms. The next section presents the new axioms. It has been shown empirically and theoretically (Posner [2] and Chichilnisky [3]) that standard tools of decision making under uncertainty are ill suited to evaluate such risks, more on this below.

In sum: the standard approaches do not provide a satisfactory answer and we provide here an alternative approach to risk management that seems better suited to the management of catastrophic risks and risks that are most likely to occur in the very distant future. This approach has an axiomatic treatment that parallels Von Neumann's theory of choice under uncertainty, but extends it requiring equal treatment for frequent and rare events.

The next section provides empirical motivation for the new approach by comparing it with expected utility in two risk profiles: asteroids and global warming risks.

## 4. Two Risk Profiles: Asteroids and Global Warming

In September 16, 2000, the British Task Force on Potentially Hazardous Near Earth Objects (NEOs) produced a report classifying asteroids risks by their size, energy yield, and average interval between impacts. Large-mass extinctions—for example, the Cretaceous Terciary Geological boundary—follow from the impact of asteroids of 16 km in diameter, which occur on the average once every 100 million years, and threaten the survival of all advanced life forms on the planet of which 99.9% have already gone extinct [18]. Below we compare the risk profile presented by asteroids with global warming risks.

 (i) An asteroid impact of this magnitude occurs on average once every 100 million years.

 (ii) It produces damage of about \$120 trillion [2], obliterating all human-produced value in the planet.

(iii) The damage is permanent—it continues annually for about 1 billion years, the expected lifetime of our planet before it is destroyed by our sun becoming a red star.

(iv) Existing observations indicate that such an impact will not take place in the next 30 years.

Below we compare this risk with the risk of "global warming" with the following simplified profile.

 (i) The probability of global warming is 1, namely, it is happening.

 (ii) The best estimate is that it will produce damage that is calculated for the catastrophic case to bring in a permanent loss of about \$2 trillion a year loss in the US and globally about \$8 trillion a year; see, for example, the study by (Posner in [2]). There is no consensus on whether the gradual or the catastrophic case for global warming is more likely.

Before examining the two risk profiles, we explain the connection between rare events and events in the distant future.

## 5. Decisions over Time and under Uncertainty

We adopt a simple approach that views "choices over time" and "choices under uncertainty" as two aspects of one and the same phenomenon.

The *probability* of an event is viewed as the *frequency* with which this event is likely to occur through time. (A word of warning is in order. This is not the only approach to defining probabilities—indeed many people object to it because of it views reality as an experiment that can repeat itself. Yet for our purposes here the approach has an important advantage in that it simplifies matters and at the same time generalizes the results.) For example, drawing "heads" with a fair coin is an event with probability 0.50 because in repeated flipping of the coin, "tails" tend to occur 50% of the time. The appearance of "heads" is thus a relatively frequent event, one that will on average occur one out of every two trials. If we flip a coin every year, for example, heads will occur almost surely in our lifetime. In this context, high frequency over time translates into high-probability and vice versa. Low-frequency events translate into low probability and vice versa. This way we treat "time" and "uncertainty". as two aspects of the same phenomenon.

We saw that high-probability events are those that occur frequently in time. In the case of asteroids, we know with certainty that at some point in time one will hit the earth and destroy most life on the planet unless we take action. The probability of such destructive event sometime in the future is one, although the event is so infrequent that the probability is essentially zero in any person's lifetime.

A *catastrophe* has been defined in [2, 6] as an event with enormous negative consequences—such as the extinction of the species—an event that may occur with probability one sometime in the distant future, but has nearly zero probability of occurring during the lifetime of any one individual. There is basically zero risk that the "catastrophe" will occur in our lifetime, although we know for sure it will occur at some point in the future. It is possible although very unlikely that we will pick up the papers tomorrow and read that some astronomer has discovered a massive comet taking deadly aim at our planet. With this definition, it becomes clear that dealing with catastrophes is the same as dealing with events that take place in the distant future. It is well known that the losses from events that take place in the very long-run future are "discounted" in an exponential manner to obtain a "present discounted value", and that this exponential discount renders long-term losses almost meaningless. Having connected "choices over time" and "choices under uncertainty", this explains why expected value, which is the parallel to "present discounted value", leads to counterintuitive evaluation of catastrophic losses or for that matter distort outcomes generally. Next we provide a formal framework.

## 6. The Mathematics of Risk

A system is in one of several possible states that can be described by real numbers. To each state $s \in R$ there is an associated outcome, so that one has $x(s) \in R^N$, $N \geq 1$. A description of outcomes across all states is called a *lottery* $x : R \rightarrow R^N$. The space of all lotteries is therefore a function space $L$ (identified in the following by the space of all essentially

bounded real-valued functions with the "sup norm", denoted $L_\infty$). Under conditions of uncertainty one makes decisions by ranking lotteries in $L$.

Von Neumann-Morgenstern (NM) axioms provided a mathematical formalization of how to rank or order lotteries. Optimization according to such an order is called *expected utility* (EU) and defines standard decision making under uncertainty. The main result from the NM axioms is that the decision procedure is obtained by optimizing a function of the following form:

$$W(x) = \int_{s \in R} u(x(s)) d\mu(x), \tag{6.1}$$

where the real line $R$ is the state space, $x : R \to R^N$ is a "lottery", $u : R^N \to R$ is a *utility function* describing the utility provided by the outcome of the lottery in each state $s$, $u(s)$, and where $d\mu(x)$ describes a countably additive measure over sets of states (or events) in $R$ that determines their relative frequency. Arrow [11] explains why utility functions must be essentially bounded so as to overcome the St. Petersburg paradox. This implies that the space $L$ of utility values provided by lotteries is $L_\infty$, namely, it is the space of measurable and (essentially) bounded functions (An essentially bounded function is a measurable function that is bounded except perhaps on a set of measure zero.) on the line $R$. Using the EU criterion, a lottery $x$ is ranked above another $y$ if and only if $W$ assigns to $x$ a larger real number:

$$x \succ y \iff W(x) > W(y), \tag{6.2}$$

where $W$ satisfies (6.1). The optimization of expected utility (EU) is a widely used procedure for evaluating choices under uncertainty. (The Euler-Lagrange equations are typically used to characterize optimal solutions.)

In the following examples we consider the space of lotteries to be the space of all continuous linear real-valued functions, $L_\infty$ with the sup norm, and the *dual* of $L_\infty$, denoted $L_\infty^*$, consists of all continuous real-valued functions on $L_\infty$. $L_\infty^*$ includes integrable functions on $R$ as well as purely finite additive measures [19–21] that are not representable by functions, for example a measure that assigns measure zero to all bounded measurable subsets of the line. Other examples are provided below. (An example of a purely finitely additive measure is the continuous linear real-valued function $\phi : L_\infty \to R$ defined by $\phi(f) = \lim_{s \to \infty} f(s)$ on all functions that have such a limit, and extended by using Hahn-Banach theorem to the rest.)

### 6.1. Catastrophic Risks

A *catastrophic risk* is a rare event leading to major widespread losses. Expected utility undervalues such risks: Chichilnisky [3, 4, 6] showed formally that using expected utility criteria underestimates catastrophic risks, and by doing so conflicts with the observed evidence of how humans evaluate such risks if they are likely to occur in their lifetimes. In order to formalize the problem we need some definitions.

*Definition 6.1.* A ranking $W : L_\infty \to R$ is said to be *insensitive to rare events* when $W(x) > W(y) \Leftrightarrow W(x') > W(y')$ for any two lotteries $x'$ and $y'$ that are obtained by modifying

arbitrarily $x$ and $y$ on any set of states $S \subset R$ with an arbitrarily small-probability $\varepsilon = \varepsilon(x, y)$. Formally,

$W$ is *insensitive to rare* events if $\forall x, y \; \exists \varepsilon = \varepsilon(x, y) : W(x) > W(y) \Leftrightarrow W(x') > W(y')$ for all $x', y'$ satisfying

$$y' = y \quad \text{a.e. on } S^c \subset R, \qquad x = x' \quad \text{a.e. on } S^c \subset R, \quad \text{where } \mu(S) < \varepsilon. \qquad (6.3)$$

Otherwise, $W$ is said to be *sensitive* to rare events.

*Definition 6.2.* $W$ is *insensitive to frequent events* if

$$\forall x, y \; \exists \varepsilon = \varepsilon(x, y) : W(x) > W(y) \Longleftrightarrow W(x') > W(y') \quad \forall x', y' \text{ s.t.}$$
$$x' = x, \quad y' = y \quad \text{a.e. on } S^c \subset R : \mu(S) > 1 - \varepsilon. \qquad (6.4)$$

Otherwise, $W$ is called *sensitive* to frequent events.

Mathematically, the problem with expected utility is that it is insensitive to rare events no matter how catastrophic these may be.

**Proposition 6.3.** *Any expected utility function* $\int_{s \in R} u(c(s)) \gamma(s) ds$, *where* $\gamma(s) \in L_1(R)$ *is insensitive to rare events.*

For a proof see the theorem in Chichilnisky [6].

## 6.2. New Axioms of Choice Under Uncertainty

Introduced in [3, 6] the following axioms contrast with Von-Neumann Morgenstern's axioms (NM) in that they treat symmetrically rare and frequent risks. They postulate that the ranking of lotteries $W : L_\infty \to R$ must satisfy the following.

*Axiom 1.* Sensitivity to rare events.

*Axiom 2.* Sensitivity to frequent events.

*Axiom 3.* Linearity and continuity of the ranking $W$.

Axioms 2 and 3 are standard; they satisfied, for example, by expected utility EU. However Axiom 1 is not satisfied by EU (Proposition 6.3 above).

To clarify the meaning of these axioms, the following are examples of rankings $W$ that do *not* satisfy our axioms.

*Example 6.4.* Consider a *criterion of choice* $W : L \to R$ that ranks lotteries assigning measure zero to any bounded set in $R$ [19–21]. Such functionals are ruled out by Axiom 2 which requires sensitivity to frequent events.

*Example 6.5.* Expected utility maximization is ruled out, as is shown in Proposition 6.3 above (see also [6]), because it does not satisfy Axiom 1.

Like the NM axioms, the new Axioms 1, 2, and 3 lead to a representation theorem.

**Theorem 6.6.** *There exist ranking criteria $\Psi : L_\infty \rightarrow R$ that satisfy all three axioms. Any criterion that satisfies the axioms is a convex combination of an expected utility plus a purely finitely additive measure focused on catastrophic events, for example:*

$$\Psi = \int_R u(c(s))\lambda(s) + \lambda(\Phi(c(s))), \tag{6.5}$$

where $c(s)$ describes the value of the lottery in state $s \in R$, $\gamma(s)$ is an integrable real-valued function on the line $R$, for example, $\lambda(s) = e^{-\delta s}$, and $u : R \rightarrow R$ is a bounded utility function.

The first term is thus an expected utility with an $L_1(R)$ density function $\gamma(s)$, and the second term is a purely finitely additive measure such as $\Phi(s) = \lim_{c \rightarrow \infty} c(s)$ for lotteries that have such a limit and extended otherwise to all lotteries by Hahn-Banach's theorem. For a proof see the study by Chichilnisky in [6]. (The optimization of functionals such as $\Psi$ is not amenable to standard tools of calculus of variations, which must be developed in new directions; see, for example, the studies by Chichilnisky [3, 6] and Heal [17].)

A recent result established that the new criterion in (6.5) is a way to formalize the notion of optimizing expected utility while bounding the worst outcome in the case of a catastrophe:

**Theorem 6.7.** *Optimizing the ranking criterion in* (6.5) *is equivalent to optimizing an expected utility function* $\int_R u(c(s))\lambda(s)$ *subject to a constraint on the possible loss in case of a catastrophe.*

For a proof see [4].

An interpretation of his theorem is as follows: the first term $\sum_{s=1}^\infty \lambda^{-s} u(x(s))$ is an integral operator with a countably additive kernel $\{\lambda^{-s}\}_{s \in Z}$ which emphasizes the weight of frequent events in the ranking of a lottery $x \in l_\infty$. The second purely finitely additive part $\Phi u(x(s))$ assigns positive weight to rare events, which have small-probability according to $\mu$. Both parts are present, so $\Psi$ is sensitive to small and to large probability events. Catastrophic risks are therefore ranked more realistically by such functionals.

Next section applies this framework for evaluating risks to the two risk profiles described above, asteroids and global warming.

## 7. Evaluating Asteroid Impacts

We first use standard methods for calculating the value of the damage caused by a risk calculated by its present value, which brings future value into the present (Posner [2] for both, and in particular page 150 for the latter). The reader should refer to the risk profile of an asteroid impact presented above.

(i) In the case of an asteroid as described above, the expected cost of the loss in any given year is obtained by multiplying \$120 trillion or \$$120 \times 10^{12}$—by the probability of the loss, which occurs on average once every 100 million years, and is therefore $10^{-8}$. Once the

loss occurs, however, it is assumed to be permanent. Therefore the expected value of the loss in year $N$ is

$$\sum_{t=1}^{\infty}\left(120 \times 10^{12} \times 10^{-8}\right) \cdot \delta^{N+t} = \left(120 \times 10^{12} \times 10^{-8} \times \delta^N\right)\sum_{t=1}^{\infty} \delta^t$$
$$= \left(120 \times 10^{12} \times 10^{-8} \times \delta^N\right)\left(\frac{\delta}{1-\delta}\right), \tag{7.1}$$

where $\delta$ is the time "discount factor", $0 < \delta < 1$, and $1 - \delta$ is the "discount rate".

If the risk does not occur in year $N$, then it can occur in year $N + 1$, and if not in year $N + 2$, $N + 3$, and so forth, and each time it occurs, it lasts permanently. Therefore the total risk is the sum of the risk of it occurring in year 30, plus the risk of it occurring in year 31, plus the risk of it occurring in year 32, and so forth, namely,

$$\left(120 \times 10^{12} \times 10^{-8} \times \delta^N\right)\left(\frac{\delta}{1-\delta}\right)\sum_{j=1}^{\infty} \delta^j = \left(120 \times 10^{12} \times 10^{-8} \times \delta^N\right)\left(\frac{\delta}{1-\delta}\right)^2. \tag{7.2}$$

At a 5% discount rate $\delta = 0.95$, and the total expected discounted value of the loss from such as asteroid is

$$120 \times 10^{12} \times 10^{-8} \times \frac{95}{100}^{30} \times \left(\frac{95/100}{1-95/100}\right)^2 = 9.2982 \times 10^7 \quad \text{or about \$92 million.} \tag{7.3}$$

At a 10% discount rate, that value is

$$120 \times 10^{12} \times 10^{-8} \times \frac{90}{100}^{30} \times 81 = 4.1204 \times 10^6 \quad \text{or about \$4 million.} \tag{7.4}$$

At a 3% discount rate, the value is

$$120 \times 10^{12} \times 10^{-8} \times \frac{97}{100}^{30} \times 32^2 =: 4.9276 \times 10^8 \quad \text{or about \$500 million.} \tag{7.5}$$

These values pale by comparison with the estimated value of other losses such as global warming, which are estimated to be in the tens of trillions, as shown below. In all cases, therefore, it appears to makes sense to allocate more funding to the global warming problem than to the problem of preventing asteroid impacts; more on this below.

### 7.1. Evaluating Global Warming

The United Nations Intergovernmental Panel on Climate Change (IPCC) found in 1996 that human-induced global warming is already occurring. There are two main scenarios for the damages that global warming will cause: (1) catastrophic global warming effects, and (2)

a more gradual buildup of damages

*Scenario 1* (Catastrophic Global Warming). The catastrophic scenario is described by Posner in [2, page 181], as follows. There is rapid increase in temperature, which produces a damage that is calculated to bring in a permanent loss of about \$2 trillion a year in the US and globally about \$8 trillion a year. Considering the risk profile already established above (cf. also [Posner, 1992, page 181]), the present discounted value of such a disaster at a 3% discount rate is

$$2 \times 10^{12} \times \frac{\delta}{1 - \delta} = 2 \times 10^{12} \times \frac{97}{3} = 6.4667 \times 10^{13} \quad \text{or about \$65 trillion.} \tag{7.6}$$

At a 5% discount rate the number is

$$2 \times 10^{12} \times \frac{95}{5} =: 3.8 \times 10^{13} \quad \text{or about \$38 trillion,} \tag{7.7}$$

and at a 10% discount rate the number is

$$2 \times 10^{12} \times 9 =: 1.8 \times 10^{13} \quad \text{or about \$18 trillion.} \tag{7.8}$$

*Scenario 2* (Gradual Buildup of Damages). In the second scenario global warming is also here today, but temperature increases slowly and its damages increase for about 100 years to reach 1% of the planet's GDP. Global GDP is calculated to be about \$120 trillion then (the same number used in the asteroid risk). After we reach maximum damage, we consider various possibilities going forward: (1) the annual damage remains the same *a perpetuity*, and (2) damages decrease slowly and disappear 100 years later. Let us compute using standard technique the present discounted value of the losses.

In the first case using a 3% discount rate we obtain

$$\sum_{i=1}^{\infty} \frac{10^{12}}{100} \times i \times \frac{97}{100}^{\,i} = 1.077\,8 \times 10^{13}, \quad \text{which is about \$10 trillion} \tag{7.9}$$

using a 5% discount rate

$$\sum_{i=1}^{\infty} \frac{10^{12}}{100} \times i \times \frac{95}{100}^{\,i} = 3.8 \times 10^{12} \quad \text{which is about \$3.8 trillion,} \tag{7.10}$$

and using a 10% discount rate,

$$\sum_{i=1}^{\infty} \frac{10^{12}}{100} \times i \times \frac{9}{10}^{\,i} =: 9.0 \times 10^{11}, \quad \text{which is about \$900 billion.} \tag{7.11}$$

In the second case, when the damage gradually decreases until it vanishes after 100 years following its maximum impact we have using a 3% discount rate

$$\left[\sum_{i=1}^{100}\frac{10^{12}}{100}\times i\times\frac{97}{100}^{i}\right]+\left[\sum_{i=2}^{100}\frac{10^{12}}{100}\times(100-(i-1))\times\frac{97}{100}^{100+i}\right]=9.7456\times10^{12}\sim\$9.7\text{ trillion},$$

(7.12)

using a 5% discount rate

$$\left[\sum_{i=1}^{100}\frac{10^{12}}{100}\times i\times\frac{95}{100}^{i}\right]+\left[\sum_{i=2}^{100}\frac{10^{12}}{100}\times(100-(i-1))\times\frac{95}{100}^{100+i}\right]=3.7506\times10^{12}\sim\$3.7\text{ trillion}$$

(7.13)

and using a 10% discount rate

$$\left[\sum_{i=1}^{100}\frac{10^{12}}{100}\times i\times\frac{9}{10}^{i}\right]+\left[\sum_{i=2}^{100}\frac{10^{12}}{100}\times(100-(i-1))\times\frac{9}{10}^{100+i}\right]=8.9993\times10^{11}\sim\$890\text{ billion}.$$

(7.14)

As was indicated above, in all cases, and with all three discount rates, 3%, 5%, and 10%, the global warming problem overwhelms in terms of present discounted values the costs involved with asteroid impacts. This is despite the fact that even in the noncatastrophic case global warming decreased GDP by a small fraction, only 1% and only after 100 years.

## 8. Comparing Global Warming and Asteroid Impacts

Using expected values we are led to believe that preventing asteroid impacts should not rank high in our policy priorities. The results from the numerical computations provided above to evaluate the risks of asteroids and global warming seem counterintuituve. How can it be that a major disturbance caused by global warming—even when we take very conservative estimated losses of less than 1% of GDP building up slowly over 100 years—overwhelm the costs associated with an asteroid impact that can plausibly lead to the extinction of the human species? The expected value of the loss caused by an asteroid that leads to extinction—is between $500 million and $92 billion, as seen above. A loss of this magnitude is smaller that of a failure of a single atomic plant—the Russians lost more than $140 billion with the accident at Chernobyl—or with the potential risks involved in global warming that is between $890 billion and $9.7 trillion. Common sense rebels against the computation we just provided. Let us use other examples for comparison. In year 2004, the profits of the 10 biggest oil companies were about $100 billion. It seems therefore unreasonable to think of losses from asteroid impacts as valued between $500 million and $92 billion. At this rate, it seems difficult to believe that we will ever do anything about averting human extinction, since current priorities will always outweighed such infrequent events, no matter how important they may be. Is there anything wrong with this argument? The alternative is to use the evaluation criterion arising from Axioms 1, 2, and 3 above. In view of the representation theorem, the next section utilizes a ranking function $\Phi$ as defined in (6.5), and to make the computation

explicit, provides a plausible number for the parameter $\mu$ that appears in the definition of $\Phi$ above.

## 9. Catastrophes and the Survival of the Species

The axioms proposed here lead us to evaluate catastrophic risks by a formula that adds to the present expected value a second term that focuses on rare events. To focus on catastrophes that involve extinction of the human species, we evaluate the cost of an event using a sum of the presented expected value plus the cost of extinction of the species. The most conservative scenario for the cost of extinction of the species (Posner [2]) is when everyone alive today dies without warning, and at probabilities that are so small that the value of a human life is computed (according to experimental evidence) at about $\$50,000$. Recall (Posner [2]) that the value of life decreases with the probability of death, so this number corresponds to events with probabilities lower that 1 in 100 million. At such small probabilities, with the current population, the species extinction event amounts to $\$600$ trillion. We may therefore assume that, in the following mathematical expression for our criterion,

$$\Psi(x) = \sum_{s=1}^{\infty} \lambda^{-s} u(x(s)) + \mu(\Phi u(x(s))), \tag{9.1}$$

the expression $\Phi u(x(s)) = \$600$ trillion.

To use this approach, we need to specify now a value for $\mu$, which is the "weight" one gives to extremely small-probability events in one's decision making. It is clear that the weight to give to the second term—which addresses the value of the low probability and thus likely distant event—is somewhat subjective. Those concerned about the long-term fate of our species may argue that we should consider it equally to the more familiar current—and likely more frequent threat like global warming. Others may argue that our capability to deal with such a threat will improve with time and therefore we should not spend too much on a threat that will most likely only occur in the distant future. For this discussion we will just note that many people would say that it is a mistake to spend money on something that will occur millions of years from now devalue the threat for an incorrect reason. They incorrectly conclude that something that occurs only every 100 million years on average will not occur in their lifetime. But it can, in fact the likelihood is the same every year.

*Example 9.1.* Valuating the Parameter $\mu$: An Illustration.

Consider the criterion $\Phi$ defined in (6.5) above. In the studies by Chichilnisky [4, 7–9], we assume that we give the catastrophic event a weight of only 1 in 100,000, namely,

$$1 - \mu = 10^{-5}, \quad \text{or equivalently} \quad \mu = 1 - 10^{-5}. \tag{9.2}$$

On this basis we can compare the global warming scenario with the asteroid collision scenario. One takes into consideration the following.

(1) Neither of the two cases of global warming—abrupt or gradual—involve human extinction.

(2) The asteroid impact considered here does involve extinction of the human species.

Under those conditions, the total cost involved in global warming is (approximately) $66 trillion at 3% discount rates, as shown above, while the total cost involved in an asteroid impact (neglecting the presented discounted value which is no larger than $500 million as shown above), is about

$$\$600 \times 10^{12} \times 10^{-5} = 6 \times 10^9 = 6 \text{ billion}. \tag{9.3}$$

Under the conditions, therefore, the yearly investment in prevention of asteroid impacts should be about 1/10 of the yearly investment in prevention of global warming, which is currently $1.7 billion (Posner [2, page 182]) leading to $170 million, while the current expenditures are instead $3.9 million, requiring therefore to be increased by a factor of about 60.

Our rational decision maker who values the future of the species and understands what probabilities really mean, could go through the following simple analysis. For any value of $\mu$ even close to one-half the expected value we have calculated makes asteroids more threatening than global warming that is attracting all the attention of policy makers and the public today. In one sense this is satisfying since we would like to believe that we would give great value to prevent our extinction. However, we used the number of US$300 trillion ($\mu = 1/2$) for the expected value and argued that it is what we should spend to defend against extinction. This does not seem intuitively correct for many reasons, not the least of which is that we would have no resources left to do anything else. The answer to this dilemma is to recognize that what we are really interested in is utility loss from extinction rather than expected value for the dollars we allocate. This view can help us achieve an intuitively pleasing answer that we should spend as much money today on defenses against extinction as can be usefully transferred into improved protection. In the case of asteroids based on current estimates many experts believe this might be only about 10 times what we are now spending which is about US$30 million dollars. This is a small number and the corrected valuation of the risk is high enough that we should need no further analysis to decide to increase our efforts now and when new opportunities become available in the future.

## 10. Conclusions

We believe that the above analysis is the beginning of a much more extensive assessment and research about our response to all kinds of catastrophic risks. Recent results provide ways to enhance our subjective judgments about the value of $\mu$, which is approximated by the marginal utility of avoiding extinction near the catastrophe, see the study by Chichilnisky in [4].

Other methods could include the application of Bayesian analysis involving experts who understand the nature of the threats as well as the correct meaning of low probability events. A Bayesian approach can be helpful to determine both the true risk profile and the most plausible utility function for the use of resources to combat a given threat. Such evaluations identify not only high expected value but also high utility. If there are very expensive things we can do to prevent the risk the the allocations of a large amount of resources may be warranted and the problem becomes more complicated. Our political leaders will need to make the more difficult choices between meeting todays' needs compared with the need to defend against distant catastrophic threats. This is not a new challenge since

we and other nations spend a significant part of our resources to defend against the threat of nuclear war or the nuclear winter that would follow it. What is new is that now we recognize that many serious threats like those arising from glaciation, asteroid impact, and biodiversity loss are unlikely to occur within our lifetimes, yet we do not want to wake up one day and find that we are facing the impact of what was an avoidable catastrophic risk. Furthermore the same type of deficiency in our approach also exists for very rare events like tsunamis and earthquakes also leading to a poor allocation of resources, as was likely the case for the 2005 Asian tsunami. This work provides a framework to address these threats in a way that agrees with our intuition. We would like to allocate resources in a way that can be useful in reducing the catastrophic threats we face.

In conclusion we offer another perspective that might also be useful for understanding why it is now that we are confronting the dilemmas. An analogy might help. Early on nobody spent a lot of money on personal insurance to protect him/herself. As we gained more knowledge of the risks we face and as we became affluent enough we decided to spend increasing amounts of money on insurance. In a similar way our species only recently has obtained the knowledge of some of the catastrophic risks we face and developed ways to cope with them. For the moment we are seriously underinsured so any way that we can do useful things to reduce our risk we should do so. Someday in the future we may be challenged as we were doing the cold war to decide between present risks and future ones.

## Acknowledgments

## References

[1] D. M. Raup, *Extinction: Bad Genes or Bad Luck?* W. W. Norton & Company, New York, NY, USA, 1st edition, 1992.

[2] R. Posner, *Catastrophes, Risk and Resolution*, Oxford University Press, Oxford, UK, 2004.

[3] G. Chichilnisky, "An axiomatic approach to choice under uncertainty with catastrophic risks," *Resource and Energy Economics*, vol. 22, no. 3, pp. 221–231, 2000.

[4] G. Chichilnisky, "Avoiding extinction: equal treatment of the present and the future," *Economics*, vol. 3, no. 2009-32, 2009.

[5] S. M. McClure, D. I. Laibson, G. Loewenstein, and J. D. Cohen, "Separate neural systems value immediate and delayed monetary rewards," *Science*, vol. 306, no. 5695, pp. 503–507, 2004.

[6] G. Chichilnisky, "Updating von Neumann Morgenstern axioms for choice under uncertainty with catastrophic risks," in *Proceedings of the Conference on Catastrophic Risks*, The Fields Institute for Mathematical Sciences, Toronto, Canada, June 1996.

[7] G. Chichilnisky, "Avoiding extinction: the future of economics," *International Journal of Green Economics*, vol. 3, no. 1, pp. 1–18, 2009.

[8] G. Chichilnisky, "The foundations of probability with black swans," *Journal of Probability and Statistics*, vol. 2010, Article ID 838240, 11 pages, 2010.

[9] G. Chichilnisky, "The foundations of statistics with black swans," *Mathematical Social Sciences*, vol. 59, no. 2, pp. 184–192, 2010.

[10] J. Von Neumann and O. Morgenstern, *Theory of Games and Economic Behavior*, Princeton University Press, Princeton, NJ, USA, 3rd edition, 1980.

[11] K. J. Arrow, *Essays in the Theory of Risk Bearing*, North-Holland, Amsterdam, The Netherlands, 1970.

[12] I. N. Herstein and J. Milnor, "An axiomatic approach to measurable utility," *Econometrica*, vol. 21, pp. 291–297, 1953.

[13] M. Allais, "The general theory of random choices in relation to the invariant cardinal utility function and the specific probability function," in *Risk Decision and Rationality*, B. R. Munier, Ed., pp. 233–289, Reidel, Dordrecht, The Netherlands, 1988.

[14] M. J. Machina, "Expected utility analysis without the independence axiom," *Econometrica*, vol. 50, no. 2, pp. 277–323, 1982.

[15] M. Machina, "Dynamic consistency and non-expected utility models of choice under uncertainty," *Journal of Economic Literature*, vol. 22, pp. 1622–1668, 1989.

[16] A. Tversky and P. Wakker, "Risk attitudes and decision weights," *Econometrica*, vol. 63, no. 6, pp. 1255–1280, 1995.

[17] G. M. Heal, *Valuing the Future*, Columbia University Press, New York, NY, USA, 2002.

[18] A. G. Sanfey, J. K. Rilling, J. A. Aronson, L. E. Nystrom, and J. D. Cohen, "The neural basis of economic decision-making in the Ultimatum Game," *Science*, vol. 300, no. 5626, pp. 1755–1758, 2003.

[19] K. Yosida and E. Hewitt, "Finitely additive measures," *Transactions of the American Mathematical Society*, vol. 72, pp. 46–66, 1952.

[20] K. Yosida, *FunctionalAnalysis*, Classics in Mathematics, Springer, New York, NY, USA, 4th edition, 1995.

[21] A. Tversky and P. Wakkler, "Risk attitudes and decision weights," *Econometrica*, vol. 63, pp. 1225–1280, 1995.