$\lambda/z^2 \in \mathbb{R}$. These surfaces may or may not be embedded, depending on the value of λ.

We now state some known facts that, taken together, further show just how closely related surfaces of constant mean curvature 1 in $\mathbb{H}^3$ are to minimal surfaces in $\mathbb{R}^3$.

1. It was shown in [Umehara and Yamada 1992] that, if f, g, and $\Sigma \setminus \{p_j\}$ are fixed and c tends to 0, the surfaces Φ of constant mean curvature c in $\mathbb{H}^3(-c^2)$ converge locally to a minimal surface in $\mathbb{R}^3$. This can be intuited from the fact that $G \to g$ as $c \to 0$ (which follows directly from equation 2–1), and hence the above first and second fundamental forms for the surfaces Φ converge to the fundamental forms for a minimal surface as $c \to 0$ (up to a sign change in II, amounting to a change of orientation). The resulting minimal surface does not necessarily have the same global topology as the surfaces of constant mean curvature c, and it may be periodic.

2. Consider the Poincaré model for $\mathbb{H}^3(-c^2)$ for $c \approx 0$. It is a round ball in $\mathbb{R}^3$ centered at the origin with Euclidean radius $1/|c|$, endowed with a complete radially symmetric metric

$$ds_c^2 = \frac{4\sum dx_i^2}{\left(1 - c^2 \sum x_i^2\right)^2}$$

of constant sectional curvature $-c^2$. Contracting this model by a factor of $|c|$, we obtain a map to the Poincaré model for $\mathbb{H}^3$. Under this mapping, surfaces of constant mean curvature c are mapped to ones of curvature 1. Thus the problem of existence of surfaces of constant mean curvature c in $\mathbb{H}^3(-c^2)$ for $c \approx 0$ is equivalent to the problem of existence of surfaces of constant mean curvature 1 in $\mathbb{H}^3$.

3. It was shown in [Rossman et al. 1997] that a minimal surface of finite total curvature in $\mathbb{R}^3$ satisfying certain conditions (these conditions are fairly general and include most known examples) can be deformed into a surface of constant mean curvature c in $\mathbb{H}^3(-c^2)$ for $c \approx 0$, so that Σ, f, and g are the same, up to a slight adjustment of the real parameters that are used to solve the period problems. By the previous item, these surfaces are equivalent to

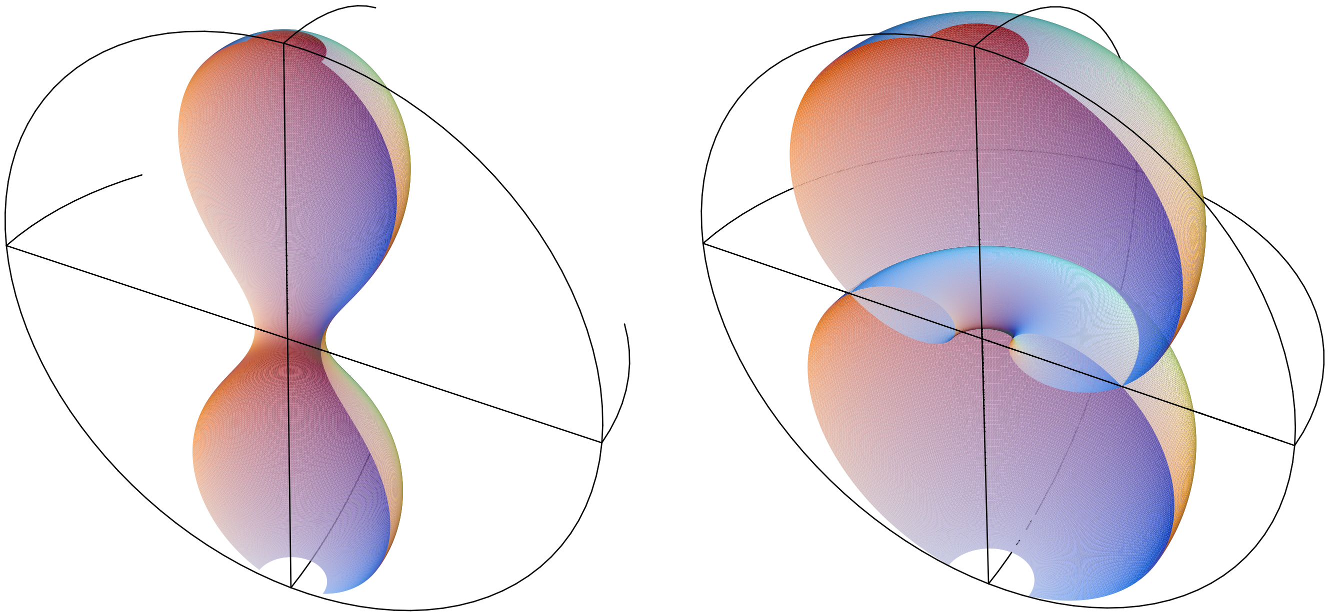

FIGURE 3. Two genus-zero catenoid cousins in the Poincaré model for $\mathbb{H}^3$. The surface on the left is embedded; the one on the right is not.