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Larger than Life: Digital Creatures in a Family of Two-Dimensional Cellular Automata

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We introduce the Larger than Life family of two-dimensional two-state cellular automata that generalize certain nearest neighbor outer totalistic cellular automaton rules to large neighborhoods. We describe linear and quadratic rescalings of John Conway's celebrated Game of Life to these large neighborhood cellular automaton rules and present corresponding generalizations of Life's famous gliders and spaceships. We show that, as is becoming well known for nearest neighbor cellular automaton rules, these "digital creatures" are ubiquitous for certain parameter values.

Keywords: Cellular automata, spaceships, Game of Life, Larger than Life

1 Introduction

John Conway's Game of Life (Life) is the most famous example of a cellular automaton (CA), due in part to the fact that its update rule is very simple yet it generates extremely complicated dynamics [BCG82], [Gar70], [GG98]. For Life, as is the case with most CA rules, the initial state has an enormous impact on the resulting dynamics. When Life is started from a random initial state with an appropriate density of occupied cells, complex structures emerge. For example, *gliders* appear and their trajectories take them across the infinite lattice (see Figure 3 on page 180). If not stopped by some other Life pattern, they will walk on forever.

The gliders are an essential ingredient for the well known result that Life is *computation universal* [BCG82]. They were also the inspiration for the *Larger than Life* (LtL) family of cellular automata. We set out to determine whether "digital creatures" analogous to Life's gliders would emerge from random initial states if we used larger neighborhoods and rescaled Life's update rule appropriately. The answer was affirmative; in fact, we found numerous rules which support digital creatures. In addition, the LtL family introduced a rich collection of two dimensional CA dynamics [Eva96].

In this paper we introduce the LtL family of rules and its digital creatures, which are called *bugs*. We describe strategies for locating rules that support bugs in two distinct regions of parameter space and give empirical evidence which suggests that the regions are connected. That is, going from one region in parameter space to the other by varying rule parameters leads us through a set of rules that also support bugs.

In addition to being interesting in their own right, some of the LtL rules we describe exhibit nonlinear population dynamics that are prototypes for various spatial models used in fields such as biology, physics, and population ecology. As one example, they might provide insights into the design of models to study the extent to which spatial variation in ecological systems influences the ability of a species to survive.

2 Larger than Life: Definition and Notation

Let us define the LtL rules. For a broader context and an account of the origins of the questions we study see [Gri94].

Each site of the *two-dimensional lattice* $\mathbf{Z}^2$ is in one of two *states*, *live* (1) or *dead* (0). This is the *initial configuration* of the system. The *neighborhood* $\mathcal{N}$ of a site consists of the $(2\rho+1)\times(2\rho+1)$ sites in the box surrounding and including it. That is, the neighborhood of the origin is $\mathcal{N}=\{y\in\mathbf{Z}^2 : ||y||_\infty \leq \rho\}$ (ρ a natural number), so that its translate $\mathcal{N}^x = x+\mathcal{N}$ is the neighborhood of site $x\in\mathbf{Z}^2$. $\mathcal{N}$ is called the *generalized Moore* or *"range* ρ*" box* neighborhood. Each *time step*, all of the sites *update* (meaning change states or not) simultaneously according to the deterministic LtL *rule*, which in words is:

- **Birth**: A site that is dead at time t will become live at time $t+1$ if and only if the number of live sites in its neighborhood at time t is in the closed interval $[\beta_1,\beta_2]$, $0\leq\beta_1\leq\beta_2$.
- **Survival**: A site that is live at time t will remain live at time $t+1$ if and only if the number of live sites in its neighborhood (itself included) at time t is in the closed interval $[\delta_1,\delta_2]$, $1\leq\delta_1\leq\delta_2$.
- **Death**: A site that is dead at time t and does not become live at time $t+1$ will remain dead at time $t+1$. A site that is live at time t and does not remain live at time $t+1$ will become dead at time $t+1$.

Let us introduce the notation needed for the precise definition of the LtL update rule and the remainder of the paper.

Let $\mathcal{T}$ denote the CA rule. That is, $\mathcal{T}:\{0,1\}^{\mathbf{Z}^2}\longmapsto\{0,1\}^{\mathbf{Z}^2}$.

Let $\xi_t(x)\in\{0,1\}$ denote the state of the site $x\in\mathbf{Z}^2$ at time t and let ξ_t represent the state of all sites in $\mathbf{Z}^2$ at time t. As is customary we will often think of the CA as a set-valued process, confounding ξ_t with $\{x:\xi_t(x)=1\}$. For example, this allows us to use the notation $\xi_t^\Lambda=\mathcal{T}^t(\Lambda)=\Lambda_t$ to mean that starting from configuration $\xi_0=\Lambda$ we arrive at the set Λ_t of occupied sites after t iterations of rule $\mathcal{T}$.

With this notation, the LtL update rule is:

$$\xi_{t+1}(x)=\begin{cases} 1 & \text{if } \xi_t(x)=0 \text{ and } |\mathcal{N}^x\cap\xi_t|\in[\beta_1,\beta_2] \\ & \text{or} \\ & \text{if } \xi_t(x)=1 \text{ and } |\mathcal{N}^x\cap\xi_t|\in[\delta_1,\delta_2]; \\ 0 & \text{otherwise.}\end{cases}$$

For each fixed range ρ the LtL CA rules form a four-parameter family indexed by the endpoints β_1 and β_2 of the birth intervals and the endpoints δ_1 and δ_2 of the survival intervals. We denote each rule by the 5-tuple $(\rho,\beta_1,\beta_2,\delta_1,\delta_2)$. In this framework Life has LtL parameters $(1,3,3,3,4)$. (Note that a live site counts itself and so the survival interval is $[3,4]$ rather than the more standard $[2,3]$.) All of the examples in this paper are from range 5; that is, $\rho=5$. The reason for this is that range 5 is big enough to give a flavor of the dynamics and local configurations from even larger ranges, yet small enough to be manageable.

The LtL CAs are *totalistic* because their update rules depend only on a site's state and the *number* of its occupied neighbors, but not on the arrangement of those neighbors [Wol94].

3 Local Space-Time Objects: Definitions and Examples

LtL's so-called digital creatures are *local* configurations. That is, they are sets of sites that are periodic under some LtL rule but that may be unstable when put on backgrounds other than all $0's$. Let us define the local configurations relevant to the remainder of this paper. These definitions arose through the study of the LtL family of rules, however, they apply to any two-state CA rules and for the most part conform to the Life terminology described in [BCG82] and [Sil]. For the following, let $\mathcal{T}$ be a CA rule from the LtL family and let $\Lambda \subset \mathbf{Z}^2$ be a set of sites in state 1. Let Λ^θ denote the configuration Λ rotated θ radians in the counterclockwise direction about the center of the smallest rectangle in which Λ can be inscribed.

- A *still life* is a configuration Λ which is a fixed point for $\mathcal{T}$. That is, $\mathcal{T}(\Lambda) = \Lambda$.
- An *oscillator* or *periodic object* is a finite configuration Λ for which there exists a positive, finite integer n so that $\mathcal{T}^t(\Lambda) = \mathcal{T}^{t+n}(\Lambda)$ for all $t \geq 0$. The smallest such n is called the *period* of Λ. For example, a *still life* is an oscillator with period 1.
- A *blinker* is an oscillator with period 2.

Example 1 *The most intriguing oscillator we have found to date has period* 166 *and is supported by the LtL rule* $(5,34,45,34,58)$. *One phase of this oscillator, denoted by* Λ *is depicted in Figure 1. As the rule updates, the oscillator cycles through its other phases, each of which is translated northeast along the diagonal until it begins a series of changes which appear to be an explosion. By time* 53 *the so-called explosion is in its last phase and results in a new configuration headed southwest. At time* 83 *the initial configuration* Λ *appears again, rotated* π *radians and translated by vector* $\vec{d} = (0,13)$. *That is,* $\mathcal{T}^{83}(\Lambda) = \Lambda^\pi + (0,13)$. *At time* 126 *another explosion begins and at time* 137 *it yields a new configuration, which is composed of a configuration headed northeast along with a disconnected piece, known as a* spark. *At time* 166 *the initial configuration* Λ *returns to its initial position on the lattice. That is,* $\mathcal{T}^{166}(\Lambda) = \Lambda$. *This oscillator is named* Bosco *and the phases of its trajectory described above are depicted in Figure 2.*

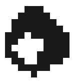

Fig. 1: Bosco: period 166 oscillator supported by LtL rule $(5,34,45,34,58)$.

- A *bug* is a finite configuration Λ for which there exists a finite time, τ, and a nonzero displacement vector, $\vec{d} = (d_1, d_2)$, such that $\mathcal{T}^\tau(\Lambda) = \Lambda + \vec{d}$. The smallest such τ is a bug's *period*, mod translation, in the direction of $\vec{d}$.

Fig. 2: Bosco's trajectory along the diagonal: heading northeast, then looping around to the southwest, and looping a final time to the northeast, to return to its initial position. Times $0, 25, 53, 83, 107$, and 137 are depicted.

- The *speed* of a bug is $max(|d_1|, |d_2|)/\tau$.
- An *orthogonal bug* is a bug whose displacement vector $\vec{d}$ has exactly one component equal to 0.
- A *diagonal bug* is a bug whose displacement vector satisfies $d_1 = d_2$ or $d_1 = -d_2$.
- A *disoriented bug* is a bug that is neither orthogonal nor diagonal.

If an orthogonal, diagonal, or disoriented bug is rotated $\theta \in \{\pi/2, \pi, 3\pi/2\}$ radians it comprises a new set which is also a bug with a trajectory perpendicular to the original or pointed in the opposite direction.

LtL's bugs are generalizations of Life's famous *spaceships*. For the uninitiated:

- A *spaceship* is any finite pattern that reappears (without additions or losses) after a number of generations and is displaced by a nonzero distance [Sil].
- A *glider* is the smallest, most common and first discovered spaceship supported by Life [Sil] (Figure 3).

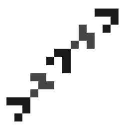

Fig. 3: The glider's trajectory starting in the southwest and moving along the diagonal to the northeast. Times 0, 11, 22, 33,and 44 are shown.

Generally, we consider bugs to be spaceships supported by rules whose neighborhoods are in ranges 2 and higher. We make this distinction because the geometry of the large range bugs is reminiscent of various insects. In some cases, as we will illustrate in Section 4.2 on page 183, the bugs are composed of line segments and appear to have *legs*. Other bugs, like those in the following examples, are composed of connected regions of $1's$ with holes in them that look like *stomachs*.

Example 2 *The LtL rule* $(5,34,45,34,58)$*, which supports the oscillator Bosco also supports orthogonal, diagonal, and disoriented bugs. Examples of such trajectories are depicted in Figures 4, 5, and 6. The geometries of the bugs' phases are reminiscent of most of Bosco's phase geometries. The question thus arises: What enables the bugs to move forward while Bosco cycles forever?*

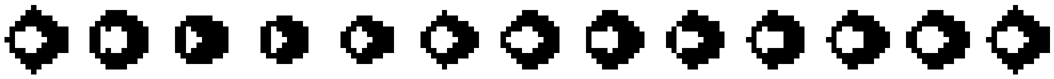

Fig. 4: Period $\tau = 12$ orthogonal bug with displacement vector $\vec{d} = (8,0)$ supported by LtL rule $(5,34,45,34,58)$. From left to right are times $23k, k = 0,1,2,...,12$ of the bug's trajectory.

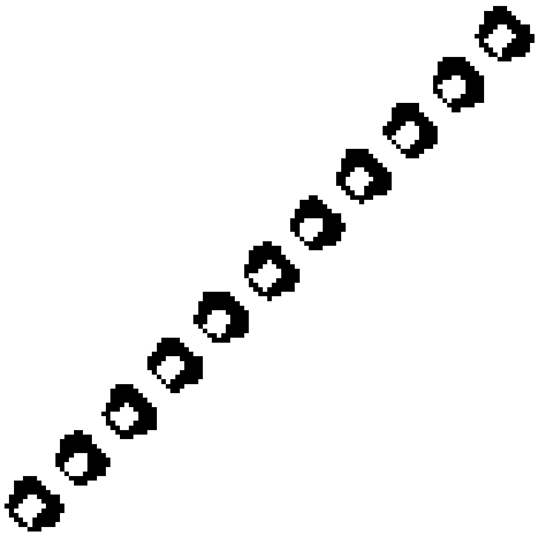

Fig. 5: Diagonal bug supported by LtL rule $(5,34,45,34,58)$, $\tau = 10$, $\vec{d} = (6,6)$. From the southwest to the northeast along the diagonal are times $17k, k = 0,1,2,...,10$ of the bug's trajectory.

The trajectories of the bug examples presented thus far have been along lines in the plane. However, some bugs, like the one in the next example, have paths that are more sinusoidal. We call such bugs "*jitter bugs*."

Example 3 *The configuration* Λ *that represents this orthogonal jitter bug is depicted in Figure 7 and its trajectory as it moves through space appears in Figure 8. The bug has period* $\tau = 66$ *and displacement vector* $\vec{d} = (40,0)$*. Compare its trajectory to the orthogonal, diagonal, and disoriented bug trajectories*

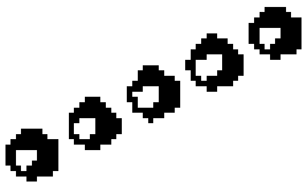

Fig. 6: Disoriented bug supported by LtL rule $(5,34,45,34,58)$, $\tau = 4$, $\vec{d} = (2,1)$. From the southwest to the northeast along the line with slope $1/2$ are times $23k, k = 0,1,2,3,4$ of the bug's trajectory.

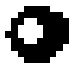

Fig. 7: Orthogonal jitter bug supported by LtL rule $(5,34,44,34,58)$, $\tau = 66$, $\vec{d} = (40,0)$.

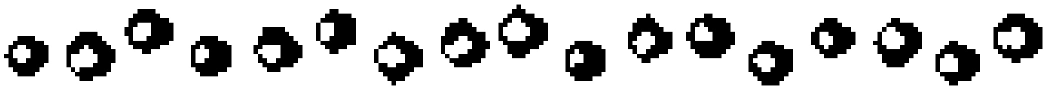

Fig. 8: From left to right are times $23k, k = 0,1,2,...,16$ of the trajectory of the orthogonal jitter bug depicted in Figure 7.

described in Example 2. The jitter bug's path may be approximated by a sine wave while the others' paths are linear.

The bugs we have seen thus far, including Life's famous glider, which is a diagonal bug with $\tau = 4$ and $\vec{d} = (4,4)$, have many phases in their evolutions as they move across the lattice. Let us define a special bug variety which has only one phase.

- An *invariant bug* is a bug for which $\tau = 1$, named such because it is invariant mod translation in the direction of $\vec{d}$.

LtL's large neighborhoods allow for more bug varieties than Life. For example, Life does not support *invariant spaceships* [Bel93] and there are no known *disoriented spaceships*, however both of these varieties commonly occur in larger range LtL rules. Due to Life's universality, disoriented spaceships that move along lines with any given rational slope could be built in theory [BCG82], however, they would likely be very large and slow. There *are* other range 1 CAs which support disoriented spaceships (see [Epp00a] for examples) and invariant spaceships. However, to date they are less common and come in fewer varieties than for rules with large ranges. A sample of range 5 LtL examples include disoriented bugs that move along lines with slopes $3/2, 2, 3, 4$ and the invariant bugs depicted in Figure 18 on page 189.

4 Navigating LtL Parameter Space in a Fixed Range

Clearly, the set of LtL rules is vast, even for a fixed range. The question thus arises: How do we navigate parameter space? Specifically, how do we locate regions in which rules support bugs? Before answering this question, we need definitions of several kinds of *global CA dynamics*. That is, limiting dynamics for the infinite system starting from a random initial state.

4.1 Global Dynamics: Definitions

Let us first present four fairly standard *quantitative* definitions.

- *Global death* is almost sure convergence to the limiting state of all $0's$. That is, $\lim_{t\to\infty}\xi_t(x) = 0$ for all $x \in \mathbf{Z}^2$, with probability one.
- ξ_t *fixates* if for each $x \in \mathbf{Z}^2$, $\xi_t(x)$ eventually has period 1 in t, with probability one. That is, $\lim_{t\to\infty}\xi_t(x)$ exists for all x, so each site changes state only finitely many times.
- ξ_t is *periodic* if for each $x \in \mathbf{Z}^2$, $\xi_t(x)$ is eventually periodic in t, with probability one. That is, for each x, there are positive finite integers n and N such that $\xi_t(x) = \xi_{t+n}(x)$ for all $t > N$.
- ξ_t generates *aperiodic dynamics* if for each $x \in \mathbf{Z}^2$, the sequence of $0's$ and $1's$ that occurs at that site (i.e. $\{\xi_t(x)\}_{t=0,1,2,\ldots}$) never cycles, with probability one. Such rules, though deterministic, behave like traditional stochastic processes.

The limiting dynamics of a rule can be quite challenging to classify. For example, Life has been studied for over 30 years, yet its classification remains elusive. Various camps claim that relaxation in Life occurs at a small exponential rate [BB91] while others argue it is *self-organized critical* [BCC89]. A third possibility is that Life supports indestructible local configurations that send out impenetrable streams of glider-like creatures and thus does not stabilize eventually. Regardless of the final answer, it is clear that Life is near some sort of "phase boundary" between aperiodic and periodic dynamics. Many of the LtL rules which support bugs are similarly challenging to classify, while others would likely be classified as aperiodic or even approaching global death. Eppstein [Epp00b] reports similar findings for nearest neighbor two-dimensional CAs. That is, he describes various range 1 rules known to support spaceships but whose global dynamics are not "Life-like." Additionally, he provides an explanation of why these findings put into question the usefulness of Wolfram's famous *universality classes* [Wol94].

Here we are interested in whether rules are capable of supporting bugs and thus introduce the following definition.

- Let *SBug* be the set of LtL rules that support at least one bug variety, regardless of their limiting dynamics.

For lack of a quantitative definition we say that a rule is "Life-like" if it supports at least one bug variety and its evolution from a random initial state is "reminiscent" of Life.

4.2 Bugs for Linear Rule Parameters

Proposition 1 *Let* $\rho \geq 2$. *The LtL rule* $(\rho, 2\rho, 2\rho, 2\rho, 2\rho)$ *is in SBug.*

Proof By definition of a *bug* we need to show that there exists a finite configuration, Λ, of $1's$, a finite time τ, and a nonzero displacement vector $\vec{d}$ such that $\mathcal{T}^{\tau}(\Lambda) = \Lambda + \vec{d}$. Let λ_1 and λ_2 be positive integers such that $\lambda_1 + \lambda_2 = 2\rho + 1$ and $2 \leq \lambda_1 \leq \lambda_2$. Let Λ be the configuration consisting of perpendicular segments, one of length λ_1 and the other of length λ_2, separated by one site (see Figure 9). Then $\mathcal{T}^2(\Lambda) =$

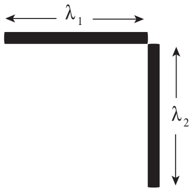

Fig. 9: Diagonal bug supported by LtL rule $(\rho, 2\rho, 2\rho, 2\rho, 2\rho)$, $\rho \geq 2$, $\tau = 2$, $\vec{d} = (1,1)$, $\lambda_1 + \lambda_2 = 2\rho + 1, 2 \leq \lambda_1 \leq \lambda_2$.

$\Lambda + (1,1)$. To see this, orient Λ so that the 0 between the two segments of $1's$ sits at the origin. Then the set that Λ comprises is $\Lambda = \{(-1,0),(-2,0),...(-\lambda_1,0),(0,-1),(0,-2),...,(0,-\lambda_2)\}$.

We check by hand to see that after one update the image of Λ under the CA rule $\mathcal{T}$ will comprise the set that is the reflection of the above set about the 45 degree line and translated by the vector $\vec{u} = (\lambda_2 - \rho, \rho - \lambda_2 + 1)$. That is, it will comprise the set $\{(0,-1),(0,-2),...,(0,-\lambda_1),(-1,0),(-2,0),...,(-\lambda_2,0)\} + \vec{u}$.

The checking we already did for Λ shows that after one update the image of the above set will comprise the set $\{(-1,0),(-2,0),...,(-\lambda_1,0),(0,-1),(0,-2),...,(0,-\lambda_2)\} + \vec{u} + \vec{w}$ where $\vec{w} = (\rho - \lambda_2 + 1, \lambda_2 - \rho)$ and this is $\Lambda + (1,1)$.

Proposition 2 *Let* $\rho \geq 3$. *The LtL rule* $(\rho, 2\rho-1, 2\rho-1, 2\rho-1, 2\rho-1)$ *is in SBug.*

In [Evab] we prove Proposition 2 and describe additional bug geometry generalizations to arbitrarily large ranges.

Based on empirical investigation we make the following:

Conjecture 1 *Let* $\rho = 2k$, $k = 3,4,5,....$ *If* $\gamma \in \{2\rho - k + 1, 2\rho - k + 1, ..., 2\rho - 2\}$ *then the LtL rule* $(\rho, \gamma, \gamma, \gamma, \gamma)$ *is in SBug.*

Conjecture 2 *Let* $\rho = 2k + 1$, $k = 2,3,4,....$ *If* $\gamma \in \{2\rho - k, 2\rho - k + 2, ..., 2\rho - 2\}$ *then the LtL rule* $(\rho, \gamma, \gamma, \gamma, \gamma)$ *is in SBug.*

According to the propositions and conjectures, as the range increases so does the number of SBug rules.

These SBug rules with parameters that are linear in the range may be considered large range versions of Life. That is, Life may be interpreted as the LtL rule $(\rho, 2\rho+1, 2\rho+1, 2\rho+1, 2\rho+2)$ for $\rho = 1$. As such, the endpoints for Life's birth and survival intervals are linear functions of the range.

In general, there are *many* more rules near those described in the propositions and conjectures which are known to support bugs. For example, in range 5 the following (nonexhaustive) set of rules are in SBug: $(5,8,8,8,j)$, $j = 9,...,14$; $(5,9,9,9,k)$, $k = 10,11,...,19$; $(5,9,9,l,l)$, $l = 3,4,...,23,25,26,28,29,,...,121$; $(5,9,9,29,m)$, $m = 30,...,121$; and $(5,10,10,10,11)$.

Let us look at the range 5 rule from Proposition 2.

Example 4 *The LtL rule* $(5,9,9,9,9)$ *supports a variety of orthogonal and diagonal bugs that emerge from an initial state composed of Bernoulli product measure with density* $1/10$ *(i.e. each site is occupied independently with probability* $1/10$*). If the density is much larger than* $1/10$ *the rule will die out immediately on a small lattice. The rule's evolution after* 9 *and* 55 *time steps is depicted in Figure 10. As seen in the figure, at time* 9 *bugs are beginning to emerge from disordered regions. By time* 55 *self-organization has taken place and only* 1 *diagonal and* 5 *orthogonal bugs remain. As the rule continues to update two* *of the bugs collide and annihilate due to periodic boundary conditions (i.e. opposite edges of the lattice are identified). In the limit,* 4 *orthogonal bugs coexist without colliding due to their relative positions on the lattice.*

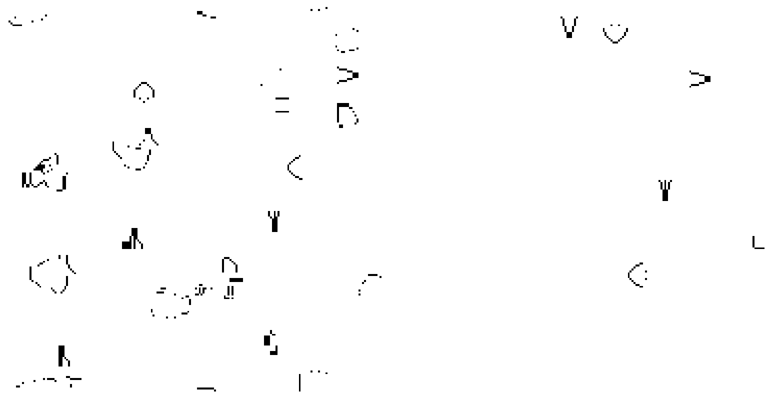

Fig. 10: Time 9 of LtL rule $(5,9,9,9,9)$ run on a 200×200 lattice with periodic boundary conditions starting from product measure with density $1/10$ is on the left. Time 55 is on the right.

Let us look more closely at the bug varieties supported by this rule, which are depicted along with their periods and displacement vectors in Figure 11. All of these bugs are representative of others supported by rules in this region, where the rule parameters are linear functions of the range. That is, the bugs are composed of line segments and look like various insects. As seen in the figure, this rule supports at least three distinct orthogonal bugs and six distinct diagonal bugs. The bugs can be used to construct various rakes*, which are objects that emit streams of bugs and are periodic mod translation in the direction of a nonzero vector* $\vec{d}$. *Some of these rakes are described in [Evaa].*

Fig. 11: Family of bugs supported by LtL rule $(5,9,9,9,9)$. Below each bug are τ and $\vec{d} = (d_1, d_2)$, the bug's period and displacement vector, respectively.

Example 5 *The LtL rule* $(5,9,9,9,17)$ *is also in SBug, however, its limiting state is quite different from the previous example. If started from a random initial state with a suitably chosen density, it eventually fixates into a maze-like pattern. To illustrate that it both supports bugs and fixates in the limit we have depicted in Figure 12 its evolution from an initially ordered state.*

All of the rules $(5,9,9,9,k), k = 10,11,...,16$ between Examples 4 and 5 are in SBug and their global dynamics would likely be described as Life-like for $k = 10,11,12,13,14$, and aperiodic for $k = 15,16$.

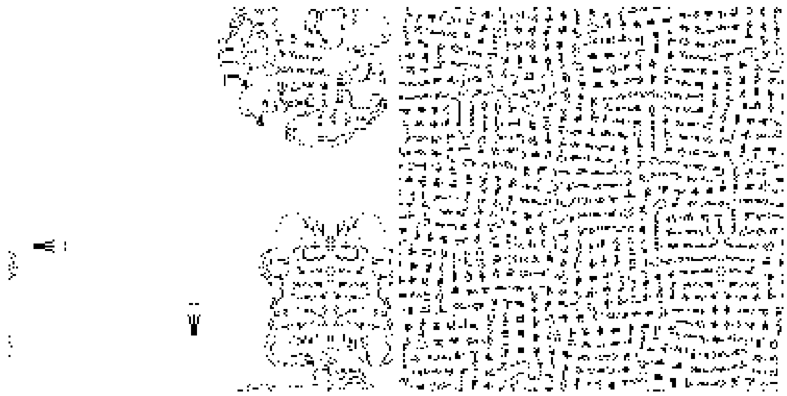

Fig. 12: Times 89 and 231 of $(5,9,9,9,17)$ started from the initially ordered state of two of the rule's period 4 orthogonal bugs and two small sets of $1's$ on a 200×200 lattice. The bugs are assimilated by time 231 into the fixed maze-like pattern of the limiting state.

The reader is encouraged to experiment with these rules using the software described in Section 5 on page 190. A bug-searching tip: as the range increases so must the lattice size, while the density of the initial state's product measure must decrease.

4.3 Bugs for Quadratic Rule Parameters

In the previous section we interpreted Life's parameters as linear functions of the range and described a region of LtL space that contains *linear Life generalizations*. Let us now present a strategy to find a region of LtL space that contains *quadratic Life generalizations*. That is, Life-like rules whose parameters are quadratic functions of the range.

The strategy requires the following mapping from range $\bar{\rho}$ LtL rules to range ρ LtL rules:

- $(\bar{\rho},\beta_1,\beta_2,\delta_1,\delta_2) \longmapsto (\rho,\lceil(\beta_1-1/2)\gamma\rceil,\lfloor(\beta_2+1/2)\gamma\rfloor,\lceil(\delta_1-1/2)\gamma\rceil,\lfloor(\delta_2+1/2)\gamma\rfloor)$, where $\gamma=((2\rho+1)/(2\bar{\rho}+1))^2$.

The mapping scales the range $\bar{\rho}$ LtL rule parameters by γ, which is *quadratic* in ρ. The following is a procedure for finding range ρ LtL rules that are in SBug:

- Fix a range $\rho \geq 2$.
- Map Life to a rule in range ρ. That is, $(1,3,3,3,4) \longmapsto (\rho,\lceil 2.5\gamma\rceil,\lfloor 3.5\gamma\rfloor,\lceil 2.5\gamma\rceil,\lfloor 4.5\gamma\rfloor)$, where $\gamma=(2\rho+1)^2/9$.
- Explore this and nearby rules.

Example 6 *The mapping from Life to range* 5 *yields the LtL rule* $(5,34,47,34,60)$. *Starting from product measure with density* $1/2$ *on a* 400×400 *lattice, this rule yields various still lifes, oscillators, and invariant bugs by time* 10. *However, these local structures are surrounded by aperiodic dynamics which quickly*

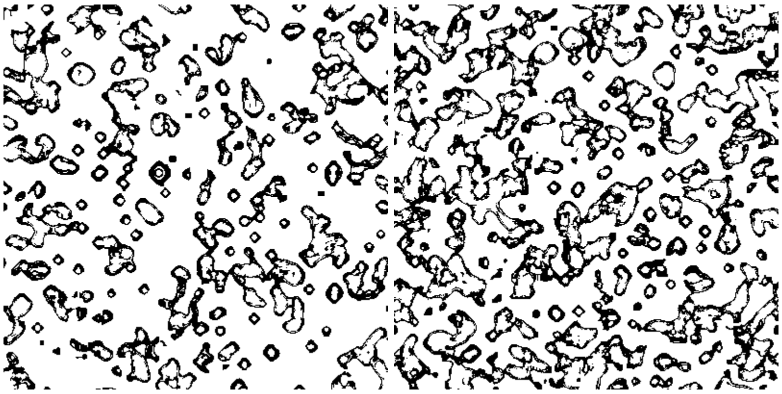

Fig. 13: The left half of the figure is time 10 of LtL rule $(5,34,47,34,60)$ run on a 400×400 lattice with periodic boundary conditions starting from product measure with density $1/2$. The right half depicts time 200.

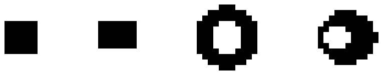

Fig. 14: Local structures supported by LtL rule $(5,34,47,34,60)$. From left to right: a still life, one phase of a blinker, one phase of a period 8 oscillator, and an invariant bug with $\vec{d}=(1,0)$.

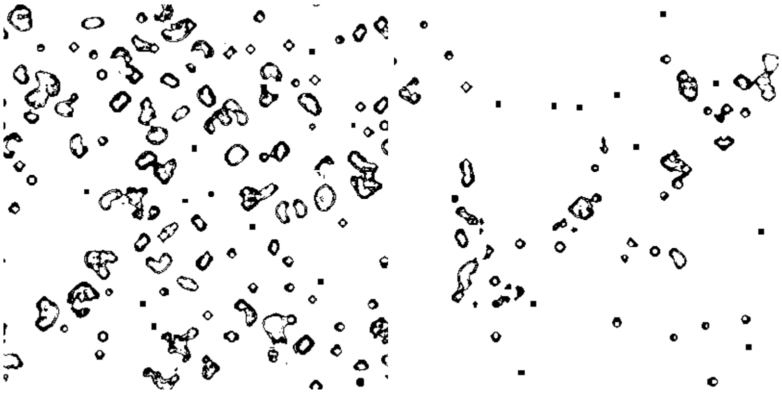

Fig. 15: The left half of the figure is time 10 of LtL rule $(5,34,45,34,58)$ run on a 400×400 lattice with periodic boundary conditions starting from product measure with density $1/2$. The right half depicts time 200.

destroys them. This evolution continues with stable local structures emerging and being destroyed. Times 10 *and* 200 *are depicted in Figure 13. The rule's invariant bug is depicted in Figure 14 along with a still life, one phase of a blinker, and one phase of a period* 8 *oscillator.*

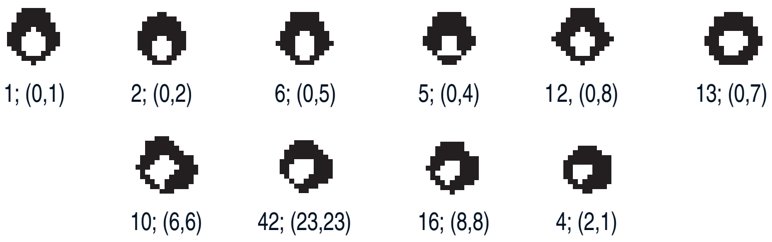

Fig. 16: Family of bugs supported by LtL rule $(5,34,45,34,58)$. Below each bug are τ and $\vec{d} = (d_1, d_2)$, the bug's period and displacement vector, respectively.

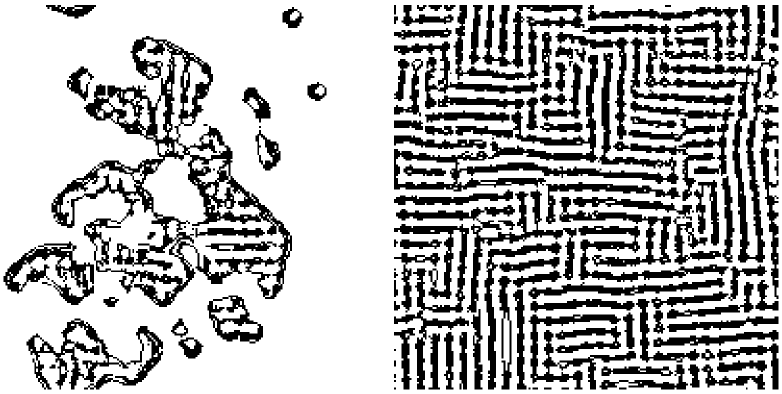

Fig. 17: Time 134 of LtL rule $(5,34,41,34,58)$ appears on the left. Its period 4 disoriented bug ($\vec{d} = (-4,-1)$) and period 11 diagonal bug ($\vec{d} = (-6,-6)$) are in the northeast corner, headed for the large growing region. By time 537, on the right, the pattern composed of approximations to vertical and horizontal stripes has assimilated the bugs and all sites of the 200×200 lattice.

Example 7 *The LtL rule* $(5,34,45,34,58)$ *supports a variety of orthogonal, diagonal, and disoriented bugs that emerge from an initial state composed of product measure with density* $1/2$*. The rule's evolution is depicted in Figure 15. As illustrated, self-organization has already taken place by time* 10 *and by time* 200 *the number of aperiodic regions has decreased. The time slices show that this rule is quite different from the nearby rule* $(5,34,47,34,60)$ *from Example 6. Both started from product measure with density* $1/2$ *on a* 400×400 *lattice with periodic boundary conditions and both support bugs and other local structures, yet this one settles down over time, while the other would likely be classified as aperiodic.*

Several of the bug varieties supported by this rule were described in Example 2 on page 181. More are depicted in Figure 16 along with their periods and displacement vectors. The rule's period 166 *oscillator named Bosco, which was described in Example 1 on page 179 can be used to construct various* bug guns*, which are stationary patterns that emit bugs forever [Sil]. Many of these are described in [Evaa].*

Example 8 *The rule* $(5,34,41,34,58)$ *is in SBug and its limiting dynamics fixates in a pattern composed of approximations to vertical and horizontal stripes. Figure 17 depicts times* 134 *and* 537 *started from an* *initially ordered state that included a period* 6 *disoriented bug* ($\vec{d} = (-4,-1)$) *and a period* 11 *diagonal bug* ($\vec{d} = (-6,-6)$)*. As illustrated in the figure, the bugs are still viable at time* 134*, but by time* 537 *they have been assimilated into the fixed pattern of the limiting state.*

There are *many* more rules near $(5,34,47,34,60)$ that are in SBug. For example, restricting our attention to rules of the form $(5,34,\beta_2,34,\delta_2)$, the following is a (nonexhaustive) set of (β_2,δ_2) values for which the rules are in SBug: $(j,60)$, $j = 45,46$; $(k,59)$, $k = 42,43,...,51$; $(l,58)$, $l = 41,42,...,53$; $(m,57)$, $m = 40,41,...,47$; $(n,56)$, $n = 39,40,...,45$.

4.4 Connecting Linear and Quadratic Life Generalizations

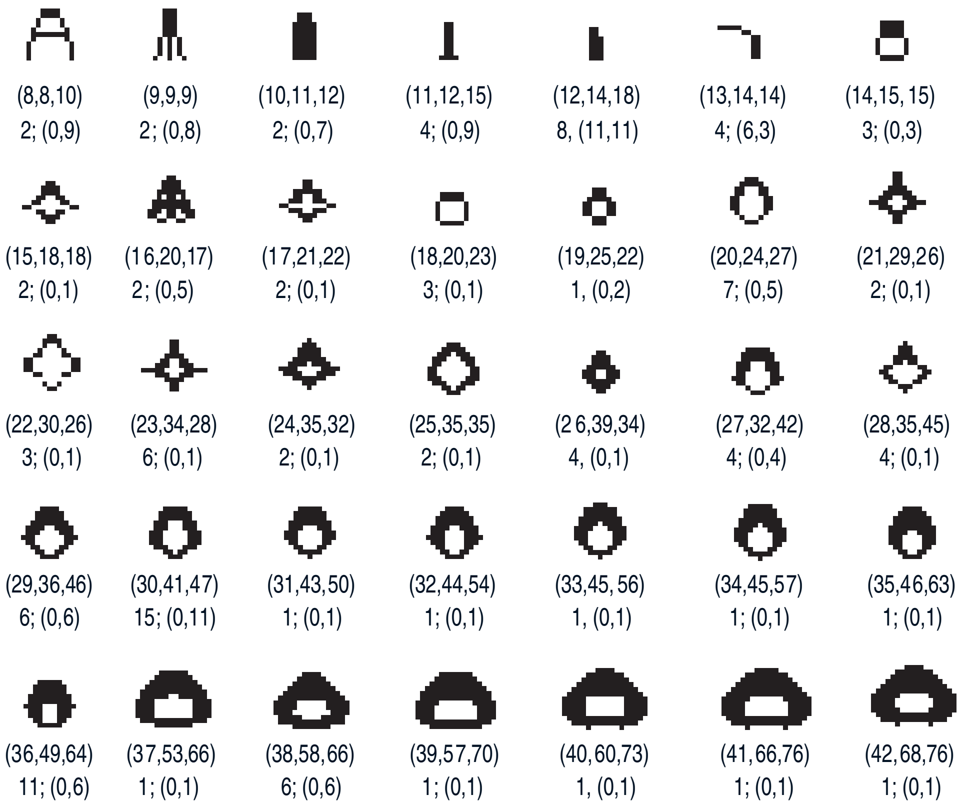

Fig. 18: Range 5 bug collection. Below each bug are the parameters $(\beta_1 = \delta_1, \beta_2, \delta_2)$ for the LtL rule that supports the bug. Also depicted are τ and $\vec{d} = (d_1, d_2)$, the bug's period and displacement vector, respectively.

We have seen that SBug rules live in distinct and distant regions of LtL parameter space. That is, there are range $\rho \geq 2$ LtL rules with $\beta_1 = \delta_1 = 2\rho$ that are in SBug and rules with $\beta_1 = \delta_1 = \lceil 2.5\gamma \rceil$ where $\gamma = (2\rho+1)^2/9$ that are in SBug. There are thus $\rho^2 - 2$ values that $\beta_1 = \delta_1$ may take on that lie "between" the distinct regions. The question thus arises: Are there SBug rules for these intermediate parameter values? In [Eva96] we show that there is a path-connected set of SBug rules between and beyond these regions in range 2 LtL parameter space. Figure 18 is empirical evidence which suggests that there is a similarly connected set of range 5 SBug rules. For each $\beta_1 = \delta_1$ from 8 to 42 the figure depicts one bug along with the parameter values for its supporting rule, its period, and its displacement vector.

The figure is only a sample of the numerous and varied range 5 bugs and their supporting rules. Using the linear rule parameters given in Section 4.2 on page 183 it is relatively easy to find SBug rules for $\beta_1 = \delta_1 = 8, 9, 10$. The rule supporting the $\beta_1 = \delta_1 = 9$ bug in the figure was discussed in detail in Example 4 on page 184. Similarly, using the procedure described in Section 4.3 on page 186, it is relatively easy to find SBug rules for $\beta_1 = \delta_1 = 27, 28, \ldots, 36$. The $\beta_1 = \delta_1 = 34$ bug in the figure is supported by several rules, including $(5, 34, 45, 34, 58)$, which was discussed in several examples. However, finding SBug rules for $\beta_1 = \delta_1 = 11, 12, \ldots, 26$ and $\beta_1 = \delta_1 = 37, 38, \ldots, 42$ is a more challenging endeavor.

A "phase transition" in *bug geometries* occurs in the fourth row of the figure and another occurs in the bottom row. A transition in rule parameter values occurs in the third row. We have yet to explore in depth the reasons for these transitions, but such a project would benefit greatly from an automated search. To this end, we plan to modify one or more of the many programs that search for spaceships supported by rules with Moore neighborhoods [Epp00b]. These sophisticated search programs might shed light on some of these questions and along the way discover new bug varieties and local configurations as yet unimagined.

5 Technology

All of the experimental work described in this paper was done using Bob Fisch and David Griffeath's *WinCA* or Mirek Wójtowicz's *MCell*, both of which are CA modeling environments for the PC and available as freeware from [Gri]. Though WinCA is the older program whose editing capabilities are modest (interaction with a paint program is necessary for creating detailed initial states) it is still superior for running large experiments from random initial states and for running LtL rules with ranges greater than 10. MCell is excellent for editing on the fly and working with small configurations. In addition a java applet is available and MCell's capabilities continue to improve.

Acknowledgements

Thanks to the Santa Fe Institute's Fellows-at-Large Program, Jim Crutchfield, and Dean Hickerson for encouraging this work and to David Griffeath for first imagining the Larger than Life CAs, also to Joan Wheeler for editorial assistance. This paper is dedicated to the memory of John Yeagley whose kindness and support will be remembered always.

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