ON A NEW METHOD OF INVESTIGATING THE RELATIONS OF SURFACES TO THEIR NORMALS, WITH RESULTS RESPECTING THE CURVATURES OF ELLIPSOIDS

By

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The important researches of Clairaut, Euler, Monge, and Dupin, on the curvatures of surfaces, leave much still to be discovered; and it appears to me that a new method of research, founded on certain new forms of equations of surfaces and of their normals, may be introduced with advantage into the subject. The method which I propose, consists in expressing, for any surface, the perpendicular distance W of an assumed origin from any tangent plane of the surface as a homogeneous function of the first dimension of the cosines α, β, γ of the inclinations of this perpendicular W to three rectangular axes of coordinates x, y, z; and in then employing the following equations of a normal:

$$x = V\alpha + \frac{\delta W}{\delta\alpha}, \quad y = V\beta + \frac{\delta W}{\delta\beta}, \quad y = V\gamma + \frac{\delta W}{\delta\gamma},$$

in which V is the length of the normal, from the surface to the point x, y, z: so that the conditions of intersection of two near normals are found by making α, β, γ vary, subject to the relation

$$\alpha\,\delta\alpha + \beta\,\delta\beta + \gamma\,\delta\gamma = 0,$$

and leaving x, y, z, and V, unchanged.

Thus for an Ellipsoid referred to its centre and axes, I use the equation

$$W = \sqrt{a^2\alpha^2 + b^2\beta^2 + c^2\gamma^2},$$

and easily obtain the following new results.

1st. The normals to an ellipsoid, for any line of one curvature have a constant sum, and for any line of the other curvature they have a constant difference, of inclinations to the normals of the two planes of circular section.

2d. The difference of the two radii of curvature at any variable point of the ellipsoid, is proportional to the product of the sines of the inclinations of the variable normal to the normals of the two circular sections, divided by the cube of the perpendicular distance of the centre from the tangent plane.

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