ADDITIONAL THEOREMS RESPECTING CERTAIN RECIPROCAL SURFACES

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By Sir William R. Hamilton.

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Sir W. R. Hamilton stated the following additional theorems respecting certain reciprocal surfaces, to which his own methods have conducted him.

If a plane quadrilateral $ABCD$ be inscribed in a given sphere, so that its four sides may be constantly parallel to four given straight lines; and if E, F be the two points of meeting of the two pairs of opposite sides, namely, E the meeting of the sides AB, CD, and F the meeting of BC, DA (prolonged if necessary); then the locus of the point E will be one ellipsoid, and the locus of the point F will be another ellipsoid reciprocal thereto.

And other pairs of reciprocal surfaces of the second degree may be generated in like manner, by changing the sphere to other surfaces of revolution of the second degree.

For instance, a pair of reciprocal cones of the second degree may be generated as the loci of two points E, F, which are, in like manner, the points of meeting of the opposite sides of a plane quadrilateral $ABCD$, inscribed in a circular section of a right-angled cone of revolution, with their directions in like manner constant. And a pair of reciprocal hyperboloids (whether of one or of two sheets) may, in like manner, be generated from an equilateral hyperboloid of revolution (of one or of two sheets).

The writer may take this opportunity of mentioning a result which lately occured to him, respecting two *arbitrary*, but *reciprocal* conical surfaces, of which each is the locus of all the normals to the others, erected at their common vertex; namely, that two such cones have always one common conical surface of centres of curvature.