ON AN EQUATION OF THE ELLIPSOID

By

William Rowan Hamilton

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By Sir William R. Hamilton.

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The Secretary of Council read the following communication from Sir William Rowan Hamilton, on an equation of the ellipsoid.

"A remark of your's, recently made, respecting the form in which I first gave to the Academy, in December, 1845, an equation of the ellipsoid by quaternions,—namely, that this form involved only *one* asymptote of the focal hyperbola,—has induced me to examine, simplify, and extend, since I last saw you, some manuscript results of mine on that subject; and the following new form of the equation, which seems to meet your requisitions, may, perhaps be shewn to the Academy tonight. This new form is the following:

$$\mathrm{TV}\frac{\eta\rho - \rho\theta}{\mathrm{U}(\eta - \theta)} = \theta^2 - \eta^2. \tag{1}$$

"The constant vectors η and θ are in the directions of the two asymptotes required; their symbolic sum $\eta + \theta$, is the vector of an umbilic; their difference, $\eta - \theta$, has the direction of a cyclic normal; another umbilicar vector being in the direction of the sum of their reciprocals, $\eta^{-1} + \theta^{-1}$, and another cyclic normal in the direction of the difference of those reciprocals, $\eta^{-1} - \theta^{-1}$. The lengths of the semiaxes of the ellipsoid are expressed as follows:

$$a = \mathrm{T}\eta + \mathrm{T}\theta; \quad b = \mathrm{T}(\eta - \theta); \quad c = \mathrm{T}\eta - \mathrm{T}\theta. \tag{2}$$

"The focal ellipse is given by the system of the two equations

$$\mathrm{S} \,.\, \rho\,\mathrm{U}\eta = \mathrm{S} \,.\, \rho\,\mathrm{U}\theta; \tag{3}$$

and

$$\mathrm{TV} \,.\, \rho\,\mathrm{U}\eta = 2\mathrm{S}\sqrt{(\eta\theta)}; \tag{4}$$

where $\mathrm{TV} \,.\, \rho\,\mathrm{U}\eta$ may be changed to $\mathrm{TV} \,.\, \rho\,\mathrm{U}\theta$; and which represent respectively a plane, and a cylinder of revolution. Finally, I shall just add what seems to me remarkable,—though I have met with several similar results in my unpublished researches,—that the focal hyperbola is adequately represented by the *single* equation following:

$$\mathrm{V} \,.\, \eta\rho \,.\, \mathrm{V} \,.\, \rho\theta = (\mathrm{V} \,.\, \eta\theta)^2.\text{"} \tag{5}$$